IV. *On the Determination of the Terms in the Disturbing Function of the fourth order as regards the Eccentricities and Inclinations which give rise to Secular Inequalities. By* J. W. Lubbock, *V.P. and Treas. R.S.*

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HITHERTO in the theory of the secular inequalities the terms in the disturbing function of the fourth order as regards the inclinations have been neglected. As the magnitude of these terms depends, in great measure, upon certain numerical coefficients, it is impossible to form any precise notion *à priori* with respect to their amount, and as to the error which may arise from neglecting them. I have therefore thought it desirable to ascertain their analytical expressions; and the details of this calculation form the subject of this paper. Some of the secular inequalities which result from these terms are far within the limits of accuracy which Laplace appears to have contemplated in the third volume of the Mécanique Céleste.

The method which I have here adopted for developing the disturbing function rests upon principles which I have already explained *. Very little trouble is requisite to obtain certain analytical expressions for the terms upon which the secular inequalities depend, or for any others, in the development of the disturbing function; but it is not so easy to put these expressions in the simplest form of which they are susceptible; and this is a point to which I think hitherto sufficient attention has not been paid. It will be found that I have obtained, finally, expressions of very remarkable simplicity: to accomplish this, however, I have been obliged to go through tedious processes of reduction, the details of which are here subjoined, in order that my results may be verified or corrected without difficulty. In order to give an additional example of the great facility with which terms in the disturbing function are arrived at by my method, I have calculated one of those given by Professor Airy, and which is required in the determination of his inequality of Venus; and I have arrived at the result which he has given. The same method, with certain modifications, is applicable to the development of the disturbing function in terms of the true longitudes. The terms in the disturbing function which give rise to the secular inequalities of the elliptic constants, when the terms of the order of the fourth powers of the eccentricities and inclinations are retained, and higher powers of those quantities are neglected, are as follows: and I propose, as they form, in fact, a system apart, to distinguish them by the indices given in the left-hand column.

* Philosophical Transactions, 1832, Part II.

o. 0
I. $\tau - \xi + \xi_{\prime}$
II. $2\tau - 2\xi + 2\xi_{\prime}$
III. $\tau + \xi + \xi_{\prime} - 2\eta$
IV. $\tau - \xi - \xi_{\prime} + 2\eta_{\prime}$
V. $\xi - \xi_{\prime} - \eta + \eta_{\prime}$
VI. $\xi + \xi_{\prime} - \eta - \eta_{\prime}$
VII. $2\tau + 2\xi_{\prime} - 2\eta$
VIII. $2\xi - 2\eta$
IX. $2\tau - 2\xi + 2\eta_{\prime}$
X. $2\xi_{\prime} - 2\eta_{\prime}$
XI. $\tau - \eta + \eta_{\prime}$
XII. $\tau - 2\xi + \eta + \eta_{\prime}$
XIII. $\tau + 2\xi_{\prime} - \eta - \eta_{\prime}$
XIV. $2\tau - \xi + \xi_{\prime} - \eta + \eta_{\prime}$
XV. $2\tau - 2\eta + 2\eta_{\prime}$.

After extensive reductions I find

$$R = -\frac{m_{\prime}}{2a_{\prime}} b_{1,0} - \frac{m_{\prime} a}{8a_{\prime}^2} b_{3,1} \left\{ e^2 + e_{\prime}^2 - \gamma^2 - \gamma_{\prime}^2 \right\} + m_{\prime} \left\{ -\frac{3a^3}{32a_{\prime}^4} b_{5,1} + \frac{3a^2}{128a_{\prime}^3} b_{5,2} \right\} e^4$$

$$-\frac{9m_{\prime}a^2}{32a_{\prime}^3} b_{5,0} e^2 e_{\prime}^2 + m_{\prime} \left\{ -\frac{3a}{32a_{\prime}^2} b_{5,1} + \frac{3a^2}{128a_{\prime}^3} b_{5,2} \right\} e_{\prime}^4$$

$$+ m_{\prime} \left\{ -\frac{3a^2}{16a_{\prime}^3} b_{5,1} + \frac{3a^2}{32a_{\prime}^3} b_{5,2} \right\} \left\{ \gamma^4 + \gamma_{\prime}^4 \right\}$$

$$+ m_{\prime} \left\{ -\frac{9a^2}{32a_{\prime}^3} b_{5,0} - \frac{3a^2}{16a_{\prime}^3} b_{5,2} \right\} \gamma^2 \gamma_{\prime}^2 + \frac{9m_{\prime}a^2}{32a_{\prime}^3} b_{5,0} \left\{ e^2 + e_{\prime}^2 \right\} \left\{ \gamma^2 + \gamma_{\prime}^2 \right\}$$

[o.]

$$+ m_{\prime} \left\{ \frac{a}{4a_{\prime}^2} b_{3,2} + \left\{ \frac{15a^2}{32a_{\prime}^3} b_{5,1} - \frac{3a}{16a_{\prime}^2} b_{5,2} \right\} e^2 + \left\{ \frac{15a^2}{32a_{\prime}^3} b_{5,1} - \frac{3a^3}{16a_{\prime}^4} b_{5,2} \right\} e_{\prime}^2 \right.$$

$$\left. + \left\{ -\frac{15a^2}{32a_{\prime}^3} b_{5,1} - \frac{3a^3}{32a_{\prime}^4} b_{5,2} \right\} \left\{ \gamma^2 + \gamma_{\prime}^2 \right\} \right\} e e_{\prime} \cos(\tau - \xi + \xi_{\prime})$$

[I.]

$$-\frac{9m_{\prime}a^2}{64a_{\prime}^3} b_{5,2} e^2 e_{\prime}^2 \cos(2\tau - 2\xi + 2\xi_{\prime}) + \frac{9m_{\prime}a^2}{32a_{\prime}^3} b_{5,1} e e_{\prime} \gamma^2 \cos(\tau + \xi + \xi_{\prime} - 2\eta)$$

[II.] [III.]

$$+\frac{9m_{\prime}a^2}{32a_{\prime}^3} b_{5,1} e e_{\prime} \gamma_{\prime}^2 \cos(\tau - \xi - \xi_{\prime} + 2\eta_{\prime}) + \frac{9m_{\prime}a^2}{16a_{\prime}^3} b_{5,1} e e_{\prime} \gamma \gamma_{\prime} \cos(\xi - \xi_{\prime} - \eta + \eta_{\prime})$$

[IV.] [V.]

$$-\frac{9m_{\prime}a^2}{16a_{\prime}^3} b_{5,1} e e_{\prime} \gamma \gamma_{\prime} \cos(\xi + \xi_{\prime} - \eta - \eta_{\prime})$$

[VI.]

$$+ m_{,} \left\{ - \frac{3\,a^3}{16\,a_{,}^4} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma^2 \cos (2\,\tau + 2\,\xi_{,} - 2\,\eta)$$

[VII.]

$$+ m_{,} \left\{ - \frac{3\,a}{16\,a_{,}^2} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} b_{5,2} \right\} e^2 \gamma^2 \cos (2\,\xi - 2\,\eta)$$

[VIII.]

$$+ m_{,} \left\{ - \frac{3\,a}{16\,a_{,}^2} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} \right\} e^2 \gamma_{,}^2 \cos (2\,\tau - 2\,\xi + 2\,\eta_{,})$$

[IX.]

$$+ m_{,} \left\{ - \frac{3\,a^2}{16\,a_{,}^3} b_{5,1} + \frac{3\,a^2}{64\,a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma_{,}^2 \cos (2\,\xi_{,} - 2\,\eta_{,})$$

[X.]

$$+ m_{,} \left\{ - \frac{a}{4\,a_{,}^2} b_{3,1} - \frac{9\,a^2}{16\,a_{,}^3} b_{5,0} (e^2 + e_{,}^2) + \frac{3\,a^2}{16\,a_{,}^3} b_{5,0} (\gamma^2 + \gamma_{,}^2) \right\} \gamma\,\gamma_{,} \cos (\tau - \eta + \eta_{,})$$

[XI.]

$$+ m_{,} \left\{ \frac{3\,a}{8\,a_{,}^2} b_{5,1} - \frac{3\,a^2}{32\,a_{,}^3} b_{5,2} \right\} e^2 \gamma\,\gamma_{,} \cos (\tau - 2\,\xi + \eta + \eta_{,})$$

[XII.]

$$+ m_{,} \left\{ \frac{3\,a^2}{8\,a_{,}^2} b_{5,0} - \frac{3\,a^2}{32\,a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma\,\gamma_{,} \cos (\tau + 2\,\xi_{,} - \eta - \eta_{,})$$

[XIII.]

$$+ m_{,} \left\{ \frac{9\,a^2}{32\,a_{,}^3} b_{5,1} + \frac{33\,a^2}{32\,a_{,}^3} b_{5,3} \right\} e\,e_{,} \gamma\,\gamma_{,} \cos (2\,\tau - \xi + \xi_{,} - \eta + \eta_{,})$$

[XIV.]

$$- m_{,} \frac{3\,a^2}{32\,a_{,}^3} b_{5,0}\, \gamma^2 \gamma_{,}^2 \cos (2\,\tau - 2\,\eta + 2\,\eta_{,})$$

[XV.]

The method which I propose to employ in order to arrive at the terms in the disturbing function, independent of the inclinations, is sufficiently explained in the Philosophical Transactions. The following are the equations employed:

$$\frac{\mathrm{d}\,R}{\mathrm{d}\,e} = \frac{a\,\mathrm{d}\,R}{\mathrm{d}\,e}\,\frac{\mathrm{d}\,r}{r\,\mathrm{d}\,e} + \frac{\mathrm{d}\,R}{\mathrm{d}\,\tau}\,\frac{\mathrm{d}\,\lambda}{\mathrm{d}\,e}$$

$$\frac{\mathrm{d}\,r}{r\,\mathrm{d}\,e} = \frac{e}{2}\left(1 + \frac{e^2}{4}\right) - \left(1 - \frac{9}{8} e^2\right) \cos \xi - \frac{3}{2} e \left(1 - \frac{11}{9} e^2\right) \cos 2\,\xi$$

[0] [2] [8]

$$- \frac{17}{8} e^2 \cos 3\,\xi - \frac{71}{24} e^3 \cos 4\,\xi$$

[20] [35]

$$\frac{\mathrm{d}\,\lambda}{\mathrm{d}\,e} = 2\left(1 - \frac{3\,e^2}{8}\right) \sin \xi + \frac{5}{2}\,e \left(1 - \frac{28}{15} e^2\right) \sin 2\,\xi + \frac{13}{4} e^2 \sin 3\,\xi + \frac{103}{24} e^2 \sin 4\,\xi.$$

[2] [8] [20] [35]

Calculation of the Term in the non-periodical Portion of R multiplied by e^4.

If R'_2 denote the term in the coefficient of $\cos\xi$ multiplied by e^3,
R_8 . . . . $\cos 2\xi$ . . . . e^2,
R'_0 . . . . non-periodical portion of R multiplied by e^2,
R_2 . . . . coefficient of $\cos\xi$ multiplied by e,

$$3R'_2 = \frac{a\,\mathrm{d}R_2}{2\,\mathrm{d}a} - \frac{a\,\mathrm{d}R_8}{2\,\mathrm{d}a} - \frac{3\,a\,\mathrm{d}R_2}{4\,\mathrm{d}a} - \frac{a\,\mathrm{d}R'_0}{\mathrm{d}a} + \frac{9\,a\,\mathrm{d}R_0}{8\,\mathrm{d}a}$$

$$R_2 = -\frac{a^2}{2\,a_i^3}\,b_{3,0} + \frac{a}{2\,a_i^3}\,b_{3,1} \qquad R_8 = -\frac{a^2}{2\,a_i^3}\,b_{3,0} + \frac{3\,a}{8\,a_i^2}\,b_{3,1}$$

$$R'_0 = -\frac{a}{8\,a_i^2}\,b_{3,1} \qquad R_0 = -\frac{1}{a_i}\,b_{1,0}$$

$$\frac{R_2}{2} - \frac{R_8}{2} - \frac{3}{4}R_2 - R'_0 + \frac{9}{8}R_0 = \frac{3\,a^2}{8\,a_i^3}\,b_{3,0} - \frac{3\,a}{16\,a_i^2}\,b_{3,1} - \frac{9}{16\,a_i}\,b_{1,0}$$

$$3R'_2 = \frac{3\,a^2}{4\,a_i^3}\,b_{3,0} - \frac{3\,a}{16\,a_i^2}\,b_{3,1} + \frac{9\,a}{16\,a_i^2}\left(\frac{a}{a_i}\,b_{3,0} - b_{3,1}\right) - \frac{9\,a^3}{8\,a_i^4}\left(\frac{a}{a_i}\,b_{5,0} - b_{5,1}\right)$$
$$+ \frac{9\,a^2}{16\,a_i^3}\left(\frac{a}{a_i}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$R'_2 = \frac{7\,a^2}{16\,a_i^3}\,b_{3,0} - \frac{a}{4\,a_i^2}\,b_{3,1} - \frac{3\,a^4}{8\,a_i^5}\,b_{5,0} + \frac{9\,a^3}{16\,a_i^4}\,b_{5,1} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}$$
$$= \frac{7\,a^2}{16\,a_i^3}\,b_{3,0} - \frac{a}{4\,a_i^2}\,b_{3,1} - \frac{3\,a^2}{8\,a_i^3}\,b_{3,0} + \frac{3\,a^2}{16\,a_i^3}\,b_{5,0} - \frac{3\,a^3}{16\,a_i^4}\,b_{5,1} + \frac{a}{16\,a_i^2}\,b_{3,1}$$
$$= \frac{a^2}{16\,a_i^3}\,b_{3,0} - \frac{3\,a}{16\,a_i^2}\,b_{3,1} + \frac{3\,a^2}{16\,a_i^3}\,b_{5,0} - \frac{3\,a^3}{16\,a_i^4}\,b_{5,1}.$$

If R''_0 denote the term in the non-periodical portion of the disturbing function multiplied by e^4,

$$4R''_0 = \frac{a\,\mathrm{d}R'_0}{2\,\mathrm{d}a} + \frac{a\,\mathrm{d}R_0}{8\,\mathrm{d}a} - \frac{a\,\mathrm{d}R'_2}{2\,\mathrm{d}a} + \frac{9\,a\,\mathrm{d}R_2}{16\,\mathrm{d}a} - \frac{3\,a\,\mathrm{d}R_8}{4\,\mathrm{d}a}$$

$$4R''_0 = \frac{1}{16\,a_i}\,b_{1,0} - \frac{a}{16\,a_i^2}\left(\frac{a}{a_i}\,b_{3,0} - b_{3,1}\right) - \frac{1}{16\,a_i}\,b_{3,0} - \frac{a}{16\,a_i^2}\,b_{3,1} + \frac{9\,a^2}{32\,a_i^3}\,b_{5,0}$$
$$- \frac{3\,a}{16\,a_i^2}\,b_{5,1} + \frac{3\,a}{16\,a_i^2}\left(\frac{a}{a_i}\,b_{5,0} - b_{5,1}\right) + \frac{3\,a^2}{32\,a_i^3}\left(\frac{a}{a_i}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$
$$- \frac{15\,a^3}{32\,a_i^4}\left(\frac{a}{a_i}\,b_{7,0} - b_{7,1}\right) + \frac{15\,a^2}{32\,a_i^3}\left(\frac{a}{a_i}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)$$
$$= \frac{1}{16\,a_i}\left(\frac{a^2 + a_i^2}{a_i^2}\,b_{3,0} - 2\,\frac{a}{a_i}\,b_{3,1}\right) - \frac{a}{16\,a_i^2}\left(\frac{a}{a_i}\,b_{3,0} - b_{3,1}\right) - \frac{1}{16\,a_i}\,b_{3,0} - \frac{a}{16\,a_i^2}\,b_{3,1}$$
$$+ \frac{9\,a^2}{32\,a_{i3}}\,b_{5,0} - \frac{3\,a}{16\,a_i^2}\,b_{5,1} + \frac{3\,a}{16\,a_i^2}\left(\frac{a}{a_i}\,b_{5,0} - b_{5,1}\right) + \frac{3\,a^2}{32\,a_i^3}\left(\frac{a}{a_i}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$- \frac{15\,a^2}{32\,a_{\prime}^3}\left(\frac{a^2+a_{\prime}^2}{a_{\prime}^2} b_{7,0} - 2\frac{a}{a_{\prime}} b_{7,1}\right) + \frac{15\,a^2}{64\,a_{\prime}^3} b_{7,0} - \frac{15\,a^2}{64\,a_{\prime}^3} b_{7,2}$$

$$= - \frac{a}{8 a_{\prime}^2}\left(\frac{a^2+a_{\prime}^2}{a_{\prime}^2} b_{5,1} - \frac{a}{a_{\prime}} b_{5,0} - \frac{a}{a_{\prime}} b_{5,2}\right) + \frac{27\,a^2}{64\,a_{\prime}^3} b_{5,0} - \frac{3\,a}{8\,a_{\prime}^2} b_{5,1}$$

$$- \frac{3\,a^2}{64\,a_{\prime}^3} b_{5,2} - \frac{15\,a^2}{32\,a_{\prime}^3} b_{5,0} + \frac{3\,a}{32\,a_{\prime}^2} b_{5,1}$$

$$= \frac{5\,a^2}{64\,a_{\prime}^3} b_{5\,0} - \frac{13\,a}{32\,a_{\prime}^2} b_{5,1} - \frac{a^3}{32\,a_{\prime}^4} b_{5,1} + \frac{5\,a^2}{64\,a_{\prime}^3} b_{5,2}$$

$$= \frac{a}{32\,a_{\prime}^2} b_{5,1} + \frac{a^3}{32\,a_{\prime}^4} b_{5,1} + \frac{a^2}{64\,a_{\prime}^3} b_{5,2} - \frac{13\,a}{32\,a_{\prime}^2} b_{5,1} - \frac{a^3}{32\,a_{\prime}^4} b_{5,1} + \frac{5\,a^2}{64\,a_{\prime}^3} b_{5,2}$$

$$= - \frac{3\,a}{8\,a_{\prime}^2} b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3} b_{5,2}.$$

Hence R contains the term

$$m_{\prime}\left\{ - \frac{3\,a}{32\,a_{\prime}^2} b_{5,1} + \frac{3\,a^2}{128\,a_{\prime}^3} b_{5,2} \right\} e^4.$$

Putting for $b_{5,1}$, $b_{5,2}$ their values in series, the first term is

$$- \frac{15\,a^2}{32\,a_{\prime}^3}.$$

This result agrees with what I have before arrived at in the Lunar theory. I have neglected no similar opportunity of verifying the terms in the disturbing function given in this paper; these opportunities are however but few, as the terms multiplied by $\gamma_{\prime}^2$ may be dispensed with in the lunar theory.

Calculation of the Term in the non-periodical Portion of the disturbing Function multiplied by $e^2\,e_{\prime}^2$.

If R'_2 now denote the term in the coefficient of cos ξ multiplied by $e\,e_{\prime}^2$,
R'_0 . . . . non-periodical portion . . . $e_{\prime}^2$,
R''_0 . . . . $e^2\,e_{\prime}^2$,

$$R'_0 = - \frac{a}{8\,a_{\prime}^2} b_{3,1} \qquad R'_2 = - \frac{a\,\mathrm{d}\,R'_0}{2\,\mathrm{d}\,a} - R'_0$$

$$R'_2 = - \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,3} - \frac{1}{2} b_{5,2}\right) + \frac{a}{3\,a_{\prime}^2} b_{3,1}$$

$$2\,R''_0 = \frac{a\,\mathrm{d}\,R'_0}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R'_2}{2\,\mathrm{d}\,a}$$

$$R'_0 - R'_2 = - \frac{a}{8\,a_{\prime}^2} b_{3,1} - \frac{a}{8\,a_{\prime}^2} b_{3,1} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2}$$

$$2\,R''_0 = - \frac{a}{8\,a_{\prime}^2} b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) + \frac{9\,a^3}{16\,a_{\prime}^4} b_{5,1}$$

$$-\frac{3a^2}{16a_i^3}b_{5,0}-\frac{3a^2}{16a_i^3}b_{5,2}-\frac{15a^4}{16a_i^5}\left(\frac{a}{a_i}b_{7,1}-\frac{1}{2}b_{7,0}-\frac{1}{2}b_{7,2}\right)$$

$$+\frac{15a^2}{32a_i^3}\left(\frac{a}{a_i}b_{7,0}-b_{7,1}\right)+\frac{15a^2}{32a_i^3}\left(\frac{a}{a_i}b_{7,2}-\frac{1}{2}b_{7,1}-\frac{1}{2}b_{7,3}\right)$$

$$=-\frac{a}{8a_i^2}b_{3,1}+\frac{15a^3}{16a_i^4}b_{5,1}-\frac{3a^2}{8a_i^3}b_{5,0}-\frac{3a^2}{4a_i^3}b_{5,2}$$

$$-\frac{15a_i^3}{16a^4}\left(\frac{a^2+a_i^2}{a_i^2}b_{7,1}-\frac{a}{a_i}b_{7,0}-\frac{a}{a_i}b_{7,2}\right)+\frac{15a^3}{64a_i^4}b_{7,1}-\frac{15a^3}{64a_i^4}b_{7,3}$$

$$=-\frac{3a^2}{16a_i^3}b_{5,0}+\frac{3a^2}{16a_i^3}b_{5,2}-\frac{3a^2}{8a_i^3}b_{5,0}-\frac{3a^2}{16a_i^3}b_{5,2}=-\frac{9a^2}{16a_i^3}b_{5,0}.$$

Hence the disturbing function contains the term

$$-\frac{9m_ia^2}{32a_i^3}b_{5,0}e^2e_i^2;\ \text{in the lunar theory}\ -\frac{9m_ia^2}{16a_i^3}e^2e_i^2.$$

Calculation of the Term in the non-periodical Portion of R multiplied by e_i^4.

If R'_5 denote the term in the coefficient of $\cos\xi_i$ multiplied by e_i^3,

R_{17} . . . . $\cos 2\xi_i$ . . . . . e_i^2,

R_5 . . . . $\cos\xi_i$ . . . . . e_i,

R'_0 . . . . non-periodical portion of R multiplied by e_i^2,

R''_0 . . . . e'^4,

$$3R'_5=\frac{a_i\,dR_5}{2\,da_i}-\frac{a_i\,dR_{17}}{2\,da_i}-\frac{3a_i\,dR_5}{4\,da_i}-\frac{a_i\,dR'_0}{da_i}+\frac{9a\,dR_0}{8\,da_i}$$

$$R_5=-\frac{1}{2a_i}b_{3,0}+\frac{a}{2a_i^2}b_{3,1}\qquad R_{17}=-\frac{1}{2a_i}b_{3,0}+\frac{3a}{8a_i^2}b_{3,1}\qquad R'_0=-\frac{a}{8a_i^2}b_{3,1}$$

$$R_0=-\frac{1}{a_i}b_{1,0}\quad\frac{R_5}{2}-\frac{R_{17}}{2}-\frac{3}{4}R_5-R'_0+\frac{9}{8}R_0=-\frac{R_5}{4}-\frac{R_{17}}{2}-R'_0+\frac{9}{8}R_0$$

$$=\frac{3}{8a_i}b_{3,0}-\frac{3a}{16a_i^2}b_{3,1}-\frac{9}{16}b_{1,0}$$

$$3R'_5=-\frac{3}{8a_i}b_{3,0}+\frac{3a}{8a_i^2}b_{3,1}+\frac{9}{16a_i}b_{1,0}-\frac{9a}{16a_i^2}\left(\frac{a}{a_i}b_{3,0}-b_{3,1}\right)$$

$$+\frac{9a}{8a_i^2}\left(\frac{a}{a_i}b_{5,0}-b_{5,1}\right)-\frac{9a^2}{16a_i^3}\left(\frac{a}{a_i}b_{5,1}-\frac{1}{2}b_{5,0}-\frac{1}{2}b_{5,2}\right)$$

$$=-\frac{3}{8a_i}b_{3,0}-\frac{9a^2}{16a_i^3}b_{3,0}+\frac{15a}{16a_i^2}b_{3,1}+\frac{9}{16a_i}\left(\frac{a^2+a_i^2}{a_i^2}b_{3,0}-\frac{2a}{a_i}b_{3,1}\right)$$

$$-\frac{9a}{16a_i^2}\left(\frac{a^2+a_i^2}{a_i^2}b_{5,1}-\frac{a}{a_i}b_{5,0}-\frac{a}{a_i}b_{5,2}\right)+\frac{27a^2}{32a_i^3}b_{6,0}-\frac{9a}{16a_i^2}b_{5,1}-\frac{9a^2}{32a_i^3}b_{5,2}$$

$$R'_5=\frac{1}{16a_i}b_{3,0}-\frac{3a}{16a_i^2}b_{3,1}+\frac{3a^2}{16a_i^3}b_{5,0}-\frac{3a}{16a_i^2}b_{5,1}$$

$$4R''_0=\frac{a_i\,dR'_0}{2\,da_i}+\frac{a_i\,dR_0}{8\,da_i}-\frac{a_i\,dR_5}{2\,da_i}+\frac{9a_i\,dR_5}{16\,da}-\frac{3a_i\,dR_{17}}{4\,da_i}$$

$$\frac{R'_0}{2} + \frac{R_0}{8} - \frac{R'_5}{2} + \frac{9}{6} R_5 - \frac{3}{4} R_{17}$$

$$= - \frac{a}{16 a_i^2} b_{3,1} - \frac{1}{16 a_i} b_{1,0} - \frac{1}{32 a_i} b_{3,0} + \frac{3 a}{32 a_i^2} b_{3,1} - \frac{3 a^2}{32 a_i^3} b_{5,0}$$

$$+ \frac{3 a}{32 a_i^2} b_{5,1} - \frac{9}{32 a_i} b_{3,0} + \frac{9 a}{32 a_i^2} b_{3,1} + \frac{3}{8 a_i} b_{3,0} - \frac{9 a}{32 a_i^2} b_{3,1}$$

$$4 R''_0 = \frac{1}{16 a_i} b_{1,0} - \frac{a}{16 a_i^2} \left(\frac{a}{a_i} b_{3,0} - b_{3,1}\right) - \frac{1}{16 a_i} b_{3,0} - \frac{a}{16 a_i^2} b_{3,1} + \frac{9 a^2}{32 a_i^3} b_{5,0}$$

$$- \frac{3 a}{16 a_i^2} b_{5,1} + \frac{3 a}{16 a_i^2} \left(\frac{a}{a_i} b_{5,0} - b_{5,1}\right) + \frac{3 a^2}{32 a_i^3} \left(\frac{a}{a_i} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$- \frac{15 a^3}{32 a_i^4} \left(\frac{a}{a_i} b_{7,0} - b_{7,1}\right) + \frac{15 a^2}{32 a_i^3} \left(\frac{a}{a_i} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right)$$

$$= \frac{1}{16 a_i} \left\{ \frac{a^2 + a_i^2}{a_i^2} b_{3,0} - 2 \frac{a}{a_i} b_{3,1} \right\} - \frac{a}{16 a_i^2} \left(\frac{a}{a_i} b_{3,0} - b_{3,1}\right) - \frac{1}{16 a_i} b_{3,0} - \frac{a}{16 a_i^2} b_{3,1}$$

$$+ \frac{9 a^2}{32 a_i^3} b_{5,0} - \frac{3 a}{16 a_i^2} b_{5,1} + \frac{3 a}{16 a_i^2} \left(\frac{a}{a_i} b_{5,0} - b_{5,1}\right) + \frac{3 a^2}{32 a_i^3} \left(\frac{a}{a_i} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$- \frac{15 a^2}{32 a_i^3} \left(\frac{a^2 + a_i^2}{a_i^2} b_{7,0} - 2 \frac{a}{a_i} b_{7,1}\right) + \frac{15 a^2}{64 a_i^3} b_{7,0} - \frac{15 a^2}{64 a_i^3} b_{7,2}$$

$$= - \frac{a}{8 a_i^2} \left(\frac{a^2 + a_i^2}{a_i^2} b_{5,1} - \frac{a}{a_i} b_{5,0} - \frac{a}{a_i} b_{5,2}\right) + \frac{27 a^2}{64 a_i^3} b_{5,0} - \frac{3 a}{8 a_i^2} b_{5,1}$$

$$- \frac{3 a^2}{64 a_i^3} b_{5,2} - \frac{15 a^2}{32 a_i^3} b_{5,0} + \frac{3 a}{32 a_i^2} b_{5,1}$$

$$= \frac{5 a^2}{64 a_i^3} b_{5,0} - \frac{13 a}{32 a_i^2} b_{5,1} - \frac{a^3}{32 a_i^4} b_{5,1} + \frac{5 a^2}{64 a_i^3} b_{5,2}$$

$$= \frac{a}{32 a_i^2} b_{5,1} + \frac{a^3}{32 a_i^4} b_{5,1} + \frac{a^2}{64 a_i^3} b_{5,2} - \frac{13 a}{32 a_i^2} b_{5,1} - \frac{a^3}{32 a_i^4} b_{5,1} + \frac{5 a^2}{64 a_i^3} b_{5,2}$$

$$= - \frac{3 a}{8 a_i^2} b_{5,1} + \frac{3 a^2}{32 a_i^3} b_{5,2}.$$

Hence the disturbing function contains the term

$$m_i \left\{ - \frac{3 a}{32 a_i^2} b_{5,1} + \frac{3 a^2}{128 a_i^3} b_{5,2} \right\} e_i^4; \text{ in the lunar theory} - \frac{15 m_i a^2}{32 a_i^3} e_i^4.$$

In the preceding instance, and in the case of terms depending either upon e^4 or e_i^4 solely, the term in the disturbing function can only be obtained from $\frac{dR}{de}$ or $\frac{dR}{de_i}$; but in obtaining those which depend *both* upon e and e_i, they may be obtained indifferently either from the combinations which enter into the expression for $\frac{dR}{de}$ or from those which enter into the expression for $\frac{dR}{de_i}$.

Calculation of the Coefficient of $e^3 e_{\prime} \cos(\tau - \xi + \xi_{\prime})$ in the Development of R.

If R_9 denote that part of the coefficient of $\cos(\tau - 2\xi)$ which is multiplied by e^2

R_1 . . . . $\cos\tau$ . . . . e^0

R'_1 . . . . e^2

R_4 . . . . $\cos(\tau + \xi)$ . . . . e

R_3 . . . . $\cos(\tau + \xi)$ . . . . e

R'_3 . . . . e^3

$$3 R'_3 = \frac{a\,\mathrm{d} R_3}{2\,\mathrm{d} a} - \frac{a\,\mathrm{d} R_9}{2\,\mathrm{d} a} + R_9 + \frac{9}{16}\frac{a\,\mathrm{d} R_1}{\mathrm{d} a} + \frac{3}{8} R_1 - \frac{3}{4}\frac{a\,\mathrm{d} R_4}{\mathrm{d} a} - \frac{5}{4} R_4 - \frac{a\,\mathrm{d} R'_1}{2\,\mathrm{d} a} - R'_1$$

$$R_3 = -\frac{3\,a}{2\,a_{\prime}^2} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,0} - \frac{a^2}{2\,a_{\prime}^2} b_{3,1} - \frac{a}{4\,a_{\prime}^2} b_{3,2} \qquad R_9 = \frac{a}{8\,a_{\prime}^2} - \frac{a}{16\,a_{\prime}^2} b_{3,0} - \frac{a}{16\,a_{\prime}^2} b_{3,2}$$

$$R_4 = \frac{a}{2\,a_{\prime}^2} - \frac{a}{4\,a_{\prime}^2} b_{3,0} - \frac{a}{2\,a_{\prime}^2} b_{3,1} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,2} \qquad R'_1 = -\frac{a}{2\,a_{\prime}^2} + \frac{a}{4\,a_{\prime}^2} b_{3,0} - \frac{a}{2\,a_{\prime}^2} b_{3,2}$$

$$R_1 = \frac{a}{a_{\prime}^2} - \frac{b_{1,2}}{a_{\prime}}\frac{R_3}{2} - \frac{R_9}{2} + \frac{9}{16} R_1 - \frac{3}{4} R_4 - \frac{R'_1}{2} = -\frac{3\,a}{8\,a_{\prime}^2} + \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{a^2}{8\,a_{\prime}^3} b_{3,1} - \frac{a}{8\,a_{\prime}^2} b_{3,2}$$

$$R_9 + \frac{3}{8} R_1 - \frac{5}{4} R_4 - R'_1 = \frac{3\,a}{8\,a_{\prime}^2} - \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{5\,a^2}{8\,a_{\prime}^3} b_{3,1} - \frac{5\,a}{16\,a_{\prime}^2} b_{3,2}$$

$$3 R_3 = -\frac{3\,a}{8\,a_{\prime}^2} + \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{a^2}{4\,a_{\prime}^2} b_{3,1} - \frac{a}{8\,a_{\prime}^2} b_{3,2} - \frac{9\,a^2}{16\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,0} - b_{5,1}\right)$$
$$- \frac{3}{8}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) - \frac{a^2}{a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3}\right)\right\}$$
$$+ \frac{3\,a}{8\,a_{\prime}^2} - \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{5\,a^2}{8\,a_{\prime}^3} b_{3,1} - \frac{5\,a}{16\,a_{\prime}^2} b_{3,2}$$
$$= \frac{7\,a^2}{8\,a_{\prime}^3} b_{3,1} - \frac{7\,a}{16\,a_{\prime}^2} b_{3,2} - \frac{9\,a^2}{16\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,0} - b_{5,1}\right)$$
$$- \frac{3}{8}\left\{\frac{a^2}{a_{\prime}^3}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{5,1} - \frac{a}{a_{\prime}} b_{5,0} - \frac{a}{a_{\prime}} b_{5,2}\right) - \frac{a^2}{2\,a_{\prime}^3} b_{5,1} + \frac{a^2}{2\,a_{\prime}^3} b_{5,3} + \frac{a^2}{2\,a_{\prime}^4} b_{5,0} - \frac{a^3}{2\,a_{\prime}^4} b_{5,2}\right\}$$
$$- \frac{3\,a^2}{8\,a_{\prime}^3} b_{3,1} - \frac{3\,a}{16\,a_{\prime}^2} b_{3,2} - \frac{9\,a^3}{16\,a_{\prime}^4} b_{5,0} + \frac{9\,a^2}{16\,a_{\prime}^3} b_{5,1}$$

$$R'_3 = -\frac{a}{16\,a_{\prime}^2} b_{3,2} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,1} - \frac{3\,a^3}{16\,a_{\prime}^4} b_{5,2}.$$

If R'_{15} denote that part of the coefficient of $\cos(\tau - \xi + \xi_{\prime})$ which is multiplied by $e_{\prime}\,e^3$,

$$R'_{15} = -\frac{a_{\prime}\,\mathrm{d} R'_3}{2\,\mathrm{d} a_{\prime}} - R'_3$$
$$= -\frac{a}{16\,a_{\prime}^2} b_{3,2} + \frac{9\,a^2}{32\,a_{\prime}^3} b_{5,1} - \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,2} + \frac{3\,a^2}{32\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3}\right)$$
$$- \frac{15}{32}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}} b_{7,2} - \frac{1}{2} b_{7,1} - \frac{1}{2} b_{7,3}\right)\right\}$$
$$+ \frac{a}{16\,a_{\prime}^2} b_{3,2} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,1} + \frac{3\,a^3}{16\,a_{\prime}^4} b_{5,2}$$

$$= \frac{3\,a^2}{64\,a_{\prime}^3} b_{5,1} - \frac{3\,a^2}{32\,a_{\prime}^3} b_{5,2} - \frac{3\,a^2}{64\,a_{\prime}^3} b_{5,3} - \frac{15}{32}\left\{ - \frac{a^3}{a_{\prime}^4}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{7,2} - \frac{a}{a_{\prime}} b_{7,1} - \frac{a}{a_{\prime}} b_{7,3}\right)\right.$$

$$\left. - \frac{a^3}{2\,a_{\prime}^4} b_{7,0} + \frac{a^3}{2\,a_{\prime}^4} b_{7,2} + \frac{a^4}{2\,a_{\prime}^5} b_{7,1} - \frac{a^4}{2\,a_{\prime}^5} b_{7,3}\right\}$$

$$= \frac{3\,a^2}{64\,a_{\prime}^3} b_{5,1} - \frac{3\,a^2}{32\,a_{\prime}^3} b_{5,2} - \frac{3\,a^2}{64\,a_{\prime}^3} b_{5,3} + \frac{15\,a^3}{32\,a_{\prime}^4} b_{5,2} + \frac{3\,a^2}{32\,a_{\prime}^3} b_{5,1} - \frac{3\,a^3}{16\,a_{\prime}^4} b_{5,2}$$

$$= \frac{15\,a^2}{32\,a_{\prime}^3} b_{5,1} - \frac{3\,a}{16\,a_{\prime}^2} b_{5,2}.$$

Hence R contains the term

$$m_{\prime}\left\{\frac{15\,a^2}{32\,a_{\prime}^3} b_{5,1} - \frac{3\,a}{16\,a_{\prime}^2} b_{5,2}\right\} e_{\prime}\, e^3 \cos(\tau - \xi + \xi_{\prime}).$$

Calculation of the Coefficient of $e\, e_{\prime}^3 \cos(\tau - \xi + \xi_{\prime})$.

If R_{19} denote that part of the coefficient of $\cos(\tau + 2\,\xi_{\prime})$ which is multiplied by $e_{\prime}^2$,

R_1 . . . . $\cos \tau$ . . . . $e_{\prime}^0$,

R'_1 . . . . $e_{\prime}^2$,

R_6 . . . . $\cos(\tau - \xi_{\prime})$ . . . . $e_{\prime}$,

R_7 . . . . $\cos(\tau + \xi_{\prime})$ . . . . $e_{\prime}$,

R'_7 . . . . $e_{\prime}^3$,

$$3\,R'_7 = \frac{a_{\prime}\,\mathrm{d}\,R_7}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\mathrm{d}\,R_{19}}{2\,\mathrm{d}\,a_{\prime}} + R_{19} + \frac{9}{16}\frac{a_{\prime}\,\mathrm{d}\,R_1}{\mathrm{d}\,a_{\prime}} + \frac{3}{8} R_1 - \frac{3}{4}\frac{a_{\prime}\,\mathrm{d}\,R_6}{\mathrm{d}\,a_{\prime}} - \frac{5}{4} R_6 - \frac{a_{\prime}\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} - R'_1$$

$$R_7 = \frac{3\,a}{4\,a_{\prime}^2} b_{3,0} - \frac{1}{2\,a_{\prime}} b_{3,1} - \frac{a}{4\,a_{\prime}^2} b_{3,2} \qquad R_{19} = \frac{a}{8\,a_{\prime}^2} - \frac{a}{16\,a_{\prime}^2} b_{3,0} - \frac{a}{16\,a_{\prime}^2} b_{3,2}$$

$$R_6 = \frac{2\,a}{a_{\prime}^2} - \frac{a}{4\,a_{\prime}^2} b_{3,0} - \frac{1}{2\,a_{\prime}} b_{3,1} + \frac{3\,a}{4\,a_{\prime}^2} b_{3,2} \qquad R'_1 = -\frac{a}{2\,a_{\prime}^2} + \frac{a}{4\,a_{\prime}^2} b_{3,0} - \frac{a}{2\,a_{\prime}^2} b_{3,2}$$

$$R_1 = \frac{a}{a_{\prime}^2} - \frac{1}{a_{\prime}} b_{1,1}$$

$$\frac{R_7}{2} - \frac{R_{19}}{2} + \frac{9}{16} R_1 - \frac{3}{4} R_6 - \frac{R'}{2} = -\frac{3\,a}{4\,a_{\prime}^2} + \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{1}{8\,a_{\prime}} b_{3,1} - \frac{a}{8\,a_{\prime}^2} b_{3,2}$$

$$R_{19} + \frac{3}{8} R_1 - \frac{5}{4} R_6 - R'_1 = -\frac{3\,a}{2\,a_{\prime}^2} - \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{5}{8\,a_{\prime}} b_{3,1} - \frac{5\,a}{16\,a_{\prime}^2} b_{3,2}$$

$$3\,R'_7 = \frac{3\,a}{2\,a_{\prime}^2} - \frac{3\,a}{8\,a_{\prime}^2} b_{3,0} - \frac{1}{8\,a_{\prime}} b_{3,1} + \frac{a}{4\,a_{\prime}^2} b_{3,2} + \frac{9\,a^2}{16\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,0} - b_{5,1}\right)$$

$$+ \frac{3}{8}\left\{\frac{a}{a_{\prime}^2}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) - \frac{a^2}{a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3}\right)\right\}$$

$$- \frac{3\,a}{2\,a_{\prime}^2} - \frac{3\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{5}{8\,a_{\prime}} b_{3,1} - \frac{5\,a}{16\,a_{\prime}^2} b_{3,2}$$

$$= -\frac{9\,a}{16\,a_{\prime}^2} b_{3,0} + \frac{1}{2\,a_{\prime}} b_{3,1} - \frac{a}{16\,a_{\prime}^2} b_{3,2} + \frac{3}{8}\left\{\frac{a}{a_{\prime}^2}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{5,2} - \frac{a}{a_{\prime}} b_{5,1} - \frac{a}{a_{\prime}} b_{5,3}\right)\right.$$

$$\left. + \frac{a^2}{2\,a_{\prime}^3} b_{5,1} - \frac{a^2}{2\,a_{\prime}^3} b_{5,3} - \frac{a}{2\,a_{\prime}^2} b_{5,0} + \frac{a}{2\,a_{\prime}^2} b_{5,2}\right\} + \frac{9\,a^3}{16\,a_{\prime}^4} b_{5,0} - \frac{9\,a^2}{16\,a_{\prime}^3} b_{5,1}$$

$$= -\frac{9\,a}{16\,a_i^2}\,b_{3,0} + \frac{1}{2\,a_i}\,b_{3,1} + \frac{a}{16\,a_i^2}\,b_{3,2} - \frac{3\,a}{8\,a_i^2}\,b_{3,2} + \frac{a}{4\,a_i^2}\,b_{3,2}$$

$$- \frac{1}{8\,a_i}\,b_{3,1} + \frac{9\,a^3}{16\,a_i^4}\,b_{5,0} - \frac{9\,a^2}{16\,a_i^3}\,b_{5,1}$$

$$= -\frac{3}{8\,a_i}\,b_{3,1} - \frac{3\,a^2}{8\,a_i^3}\,b_{3,1} + \frac{3\,a}{16\,a_i^2}\,b_{3,2} - \frac{3\,a}{16\,a_i^2}\,b_{3,2} + \frac{3}{8\,a_i}\,b_{3,1} + \frac{9\,a^3}{16\,a_i^4}\,b_{5,0} - \frac{9\,a^2}{16\,a_i^3}\,b_{5,1}$$

$$= -\frac{9\,a^2}{16\,a_i^3}\,b_{5,1} + \frac{9\,a^3}{16\,a_i^4}\,b_{5,2}.$$

$$R_7 = -\frac{3\,a^2}{16\,a_i^3}\,b_{5,1} + \frac{3\,a^3}{16\,a_i^4}\,b_{5,2}$$

If R'_{15} now denote that part of the coefficient of $\cos(\tau - \xi + \xi_i)$ which is multiplied by $e\,e_i^3$,

$$R'_{15} = -\frac{a\,\mathrm{d}\,R_7}{2\,\mathrm{d}\,a} - R_7$$

$$= \frac{3\,a^2}{16\,a_i^3}\,b_{5,1} - \frac{9\,a^3}{32\,a_i^4}\,b_{5,2} + \frac{15\,a^3}{32\,a_i^4}\,b_{5,2} + \frac{3\,a_i^2}{32\,a_i^3}\,b_{5\,1}$$

$$- \frac{3\,a^3}{16\,a_i^4}\,b_{5\,2} + \frac{3\,a^2}{16\,a_i^3}\,b_{5,1} - \frac{3\,a^3}{16\,a_i^4}\,b_{5,2}$$

$$= \frac{15\,a^2}{32\,a_i^3}\,b_{5,1} - \frac{3\,a^3}{16\,a_i^4}\,b_{5,2}$$

Hence R contains the term

$$m_i \left\{ \frac{15\,a^2}{32\,a_i^3}\,b_{5,1} - \frac{3\,a^3}{16\,a_i^4}\,b_{5,2} \right\} e\,e_i^3 \cos(\tau - \xi + \xi_i).$$

I have found that the disturbing function contains the term

$$-\frac{9\,m_i\,a^2}{64\,a_i^3}\,b_{5,2}\,e^2\,e_i^2 \cos(2\,\tau - 2\,\xi + 2\,\xi_i).$$

As I have given elsewhere the details of the calculation of this term, it is unnecessary to repeat them here.

In order to obtain the terms depending partly upon γ^2, γ_i^2, &c., from the same equation, the following transformations are necessary:

$$\cos\lambda' = \cos(\lambda' - \nu + \nu) = \cos(\lambda' - \nu)\cos\nu - \sin(\lambda' - \nu)\sin\nu$$

$$\tan(\lambda' - \nu) = \cos\iota\tan(\lambda - \epsilon)$$

$$\cos(\lambda' - \nu) = \frac{1}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}} \qquad \sin(\lambda' - \nu) = \frac{\cos\iota\tan(\lambda - \epsilon)}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}}$$

$$\cos\lambda' = \frac{\cos\nu - \cos\iota\tan(\lambda - \epsilon)\sin\nu}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}} = \frac{\cos^2\frac{\iota}{2}\cos(\lambda - \epsilon + \nu) + \sin^2\frac{\iota}{2}\cos(\lambda - \epsilon - \nu)}{\sqrt{1 - \sin^2\iota\sin^2(\lambda - \epsilon)}}$$

$$\sin\lambda' = \frac{\cos\iota\tan(\lambda - \epsilon)\cos\nu - \sin\nu}{\sqrt{1 + \cos^2\iota\tan^2(\lambda - \epsilon)}} = \frac{\cos^2\frac{\iota}{2}\sin(\lambda - \epsilon + \nu) - \sin^2\frac{\iota}{2}\sin(\lambda - \epsilon - \nu)}{\sqrt{1 - \sin^2\iota\sin^2(\lambda - \epsilon)}}$$

$$r' = r\sqrt{1 - \sin^2\iota\sin^2(\lambda - \epsilon)}$$

$$r' r'_{,} \{\cos(\lambda' - \lambda_{,}) + s s_{,}\} = r r_{,} \Big\{ \cos^2 \tfrac{\iota}{2} \cos^2 \tfrac{\iota_{,}}{2} \cos(\lambda - \lambda_{,} - \varepsilon + \varepsilon_{,} + \nu - \nu_{,})$$

$$+ \sin^2 \tfrac{\iota}{2} \cos \tfrac{\iota_{,}}{2} \cos(\lambda + \lambda_{,} - \varepsilon - \varepsilon_{,} - \nu + \nu_{,})$$

$$+ \cos^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\lambda + \lambda_{,} - \varepsilon + \nu - \nu_{,})$$

$$+ \sin^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\lambda - \lambda_{,} - \varepsilon + \varepsilon_{,} - \nu + \nu_{,})$$

$$+ \frac{\tan\iota \tan\iota_{,}}{2} \cos(\lambda - \lambda_{,} - \varepsilon + \varepsilon_{,})$$

$$- \frac{\tan\iota \tan\iota_{,}}{2} \cos(\lambda + \lambda_{,} - \varepsilon + \varepsilon_{,}) \Big\}$$

If $\tan\iota = \gamma$, $\cos^2 \frac{\iota}{2} = 1 - \frac{\gamma^2}{4} + \frac{3}{16}\gamma^4$ $\sin^2 \frac{\iota}{2} = \frac{\gamma^2}{4} - \frac{3}{16}\gamma^4$.

If $n t - n_{,} t + \varepsilon - \varepsilon_{,} - \varepsilon + \varepsilon_{,} + \nu - \nu_{,}$ be called τ, and if $n t + \varepsilon - \varepsilon = \eta$ $n_{,} t + \varepsilon_{,} - \varepsilon_{,} = \eta_{,}$ since when the eccentricities are neglected $\lambda = n t + \varepsilon$, $\lambda_{,} = n_{,} t + \varepsilon_{,}$, $r = a$, $r_{,} = a_{,}$

$$r' r'_{,} \{\cos(\lambda' - \lambda'_{,}) + s s_{,}\} = a a_{,} \Big\{ \cos^2 \tfrac{\iota}{2} \cos^2 \tfrac{\iota_{,}}{2} \cos\tau + \sin^2 \tfrac{\iota}{2} \cos^2 \tfrac{\iota_{,}}{2} \cos(\tau - 2\eta)$$

$$+ \cos^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\tau + 2\eta_{,}) + \sin^2 \tfrac{\iota}{2} \sin^2 \tfrac{\iota_{,}}{2} \cos(\tau - 2\eta + 2\eta_{,})$$

$$+ \frac{\gamma\gamma_{,}}{2} \cos(\eta - \eta_{,}) - \frac{\gamma\gamma_{,}}{2} \cos(\eta + \eta_{,})$$

$$= a a_{,} \Big\{ \left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4} + \frac{3}{16}\gamma^4 + \frac{\gamma^2\gamma_{,}^2}{16} + \frac{3}{16}\gamma_{,}^4\right) \cos\tau$$

$$+ \frac{\gamma^2}{4}\left(1 - \frac{\gamma_{,}^2}{4}\right) \cos(\tau - 2\eta) + \frac{\gamma_{,}^2}{4}\left(1 - \frac{\gamma^2}{4}\right) \cos(\tau + 2\eta_{,})$$

$$+ \frac{\gamma^2\gamma_{,}^2}{16} \cos(\tau - 2\eta + 2\eta_{,}) + \frac{\gamma\gamma_{,}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4}\right) \cos(\eta - \eta_{,})$$

$$- \frac{\gamma\gamma_{,}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{,}^2}{4}\right) \cos(\eta + \eta_{,}) \Big\}.$$

In order to have the terms required depending upon the squares of the inclinations, it is sufficient to take

$$R = -\frac{m_{,}}{a_{,}} \Big\{ \frac{1}{2} b_{1,0} + b_{1,1} \cos\tau + b_{1,2} \cos 2\tau + \&c. \Big\}$$

$$+ \frac{m_{,} a}{2 a_{,}^2} \Big\{ \frac{1}{2} b_{3,0} + b_{3,1} \cos\tau + b_{3,2} \cos 2\tau + \&c. \Big\}$$

$$\Big\{ \left(\frac{\gamma^2 + \gamma_{,}^2}{2}\right) \cos\tau - \frac{\gamma^2}{2} \cos(\tau - 2\eta) - \frac{\gamma_{,}^2}{2} \cos(\tau + 2\eta_{,})$$

$$- \gamma\gamma_{,} \cos(\eta - \eta_{,}) + \gamma\gamma_{,} \cos(\eta + \eta_{,}) \Big\}$$

$$= -\frac{m_{\prime}}{a}\left\{\frac{1}{2}b_{1,0} + b_{1,1}\cos\tau + b_{1,2}\cos 2\tau + \&c. \right\}$$

$$+ \frac{m_{\prime} a}{8 a_{\prime}^2}(\gamma^2 + \gamma_{\prime}^2) b_{3,1} + \frac{m_{\prime} a}{8 a_{\prime}^2}(b_{3,0} + b_{3,2})(\gamma^2 + \gamma_{\prime}^2)\cos\tau$$

$$- \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,0}\gamma^2 \cos(\tau - 2\eta) - \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,0}\gamma_{\prime}^2 \cos(\tau + 2\eta_{\prime})$$

$$- \frac{m_{\prime} a}{4 a_{\prime}^2} b_{3,0}\gamma\gamma_{\prime}\cos(\eta - \eta_{\prime}) + \frac{m_{\prime} a}{4 a_{\prime}^2} b_{3,0}\cos(\eta + \eta_{\prime}) - \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,1}\gamma_{\prime}^2\cos(2\tau - 2\eta)$$

$$- \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,1}\gamma^2\cos 2\eta - \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,1}\gamma_{\prime}^2\cos(2\tau + 2\eta_{\prime}) - \frac{m_{\prime} a}{8 a_{\prime}^2} b_{3,1}\gamma_{\prime}^2\cos 2\eta_{\prime}$$

$$- \frac{m_{\prime} a}{4 a_{\prime}^2} b_{3,1}\gamma\gamma_{\prime}\cos(\tau - \eta + \eta_{\prime}) + \frac{m_{\prime} a}{4 a_{\prime}^2} b_{3,1}\gamma\gamma_{\prime}\cos(\tau + \eta + \eta_{\prime})$$

$$+ \frac{m_{\prime} a}{4 a_{\prime}^2} b_{3,1}\gamma\gamma_{\prime}\cos(\tau - \eta - \eta_{\prime}) - \frac{m_{\prime} a}{4 a_{\prime}^2} b_{3,2}\gamma\gamma_{\prime}\cos(2\tau - \eta + \eta_{\prime}) + \&c.$$

As before, $\frac{\mathrm{d}R}{\mathrm{d}e} = \frac{\mathrm{d}R}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}e} + \frac{\mathrm{d}R}{\mathrm{d}\lambda}\frac{\mathrm{d}\lambda}{\mathrm{d}e}$,

but in this form of development

$$\frac{\mathrm{d}R}{\mathrm{d}\lambda} = \frac{\mathrm{d}R}{\mathrm{d}\tau} + \frac{\mathrm{d}R}{\mathrm{d}\eta} \qquad \frac{\mathrm{d}R}{\mathrm{d}\lambda_{\prime}} = -\frac{\mathrm{d}R}{\mathrm{d}\tau} + \frac{\mathrm{d}R}{\mathrm{d}\eta_{\prime}}.$$

Calculation of the Term in R multiplied by $e^2\gamma^2$.

$$R_2 = -\frac{a\,\mathrm{d}R_0}{\mathrm{d}a} \qquad R_0 = \frac{a}{8 a_{\prime}^2} b_{3,1}$$

$$R_2 = -\frac{a}{8 a_{\prime}^2} b_{3,1} + \frac{3 a^2}{8 a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$= -\frac{3 a^2}{16 a_{\prime}^3} b_{5,0} + \frac{3 a^2}{16 a_{\prime}^3} b_{5,2} + \frac{3 a^3}{8 a_{\prime}^4} b_{5,1} - \frac{3 a^2}{16 a_{\prime}^3} b_{5,0} - \frac{3 a^2}{16 a_{\prime}^3} b_{5,2}$$

$$= -\frac{3 a^2}{8 a_{\prime}^3} b_{5,0} + \frac{3 a^3}{8 a_{\prime}^4} b_{5,1}.$$

If the term in R_0 multiplied by $e^2\gamma^2$ be called R''_0,

$$2 R''_0 = \frac{a\,\mathrm{d}R_0}{2\,\mathrm{d}a} + \frac{a\,\mathrm{d}R_2}{2\,\mathrm{d}a}$$

$$= \frac{3 a^2}{16 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{16 a_{\prime}^4} b_{5,1} + \frac{3 a^2}{8 a_{\prime}^3} b_{5,0} - \frac{9 a^3}{16 a_{\prime}^4} b_{5,1}$$

$$- \frac{15}{16}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}} b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right)\right\}$$

$$= \frac{9 a^2}{16 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{4 a_{\prime}^4} b_{5,1} - \frac{15}{16}\left\{-\frac{a^3}{a_{\prime}^4}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{7,1} - \frac{a}{a_{\prime}} b_{7,0} - \frac{a}{a_{\prime}} b_{7,2}\right) + \frac{a^4}{2 a_{\prime}^5} b_{7,0} - \frac{a^4}{2 a_{\prime}^5} b_{7,2}\right\}$$

$$= \frac{9 a^2}{16 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{4 a_{\prime}^4} b_{5,1} + \frac{15 a^3}{16 a_{\prime}^4} b_{5,1} - \frac{3 a^3}{16 a_{\prime}^4} b_{5,1}$$

$$= \frac{9 a^2}{16 a_{\prime}^3} b_{5,0}.$$

Hence R contains the term $\frac{9 m_{\prime} a^2}{32 a_{\prime}^3} b_{5,0} e^2 (\gamma^2 + \gamma_{\prime}^2)$. Putting for $b_{5,0}$ its value in series according to powers of $\frac{a}{a_{\prime}}$, neglecting $\gamma_{\prime}^2$, I find for the lunar theory $\frac{9 m_{\prime} a^2}{16 a_{\prime}^3} e^2 \gamma^2$, which agrees with the result I have arrived at elsewhere by other methods.

Calculation of the Term in R_0 multiplied by $e_{\prime}^2 \gamma^2$.

$$R_5 = - \frac{a_{\prime} \mathrm{d} R_0}{\mathrm{d} a_{\prime}} \qquad R_0 = \frac{a}{8 a_{\prime}^2} b_{3,1}$$

$$R_5 = \frac{a}{4 a_{\prime}^2} b_{3,1} - \frac{3 a^2}{8 a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2} \right)$$

$$= \frac{a}{4 a_{\prime}^2} b_{3,1} - \frac{a}{8 a_{\prime}^2} b_{3,1} + \frac{3 a^2}{16 a_{\prime}^2} b_{5,0} - \frac{3 a^2}{16 a_{\prime}^3} b_{5,2} - \frac{3 a^3}{8 a_{\prime}^4} b_{5,1} + \frac{3 a^2}{16 a_{\prime}^3} b_{5,0} + \frac{3 a^2}{16 a_{\prime}^3} b_{5,2}$$

$$= \frac{a}{8 a_{\prime}^2} b_{3,1} + \frac{3 a^2}{8 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{8 a_{\prime}^4} b_{5,1}.$$

If R''_0 denote the term in R_0 multiplied $e_{\prime}^2 \gamma_{\prime}^2$,

$$2 R''_0 = \frac{a_{\prime} \mathrm{d} R_0}{2 \mathrm{d} a_{\prime}} - \frac{a_{\prime} \mathrm{d} R_5}{2 \mathrm{d} a_{\prime}}$$

$$= - \frac{a}{16 a_{\prime}^2} b_{3,1} - \frac{3 a^2}{16 a_{\prime}^3} b_{5,0} + \frac{3 a^3}{16 a_{\prime}^4} b_{5,1} + \frac{a}{8 a_{\prime}^2} b_{3,1} + \frac{9 a^2}{16 a_{\prime}^3} b_{5,0} - \frac{3 a^3}{4 a_{\prime}^4} b_{5,1}$$

$$- \frac{3 a^2}{16 a_{\prime}^3} \left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{,25} \right) + \frac{15 a^3}{16 a_{\prime}^4} b_{5,1} - \frac{3 a^3}{16 a_{\prime}^4} b_{5,1}$$

$$= \frac{9 a^2}{16 a_{\prime}^3} b_{5,0}.$$

Hence R contains the term $\frac{9 m_{\prime} a^2}{32 a_{\prime}^3} b_{5,0} e_{\prime}^2 (\gamma^2 + \gamma_{\prime}^2)$, or in the lunar theory $\frac{9 m_{\prime} a^2}{16 a_{\prime}^3} e_{\prime}^2 \gamma^2$.

Calculation of the Term in R_{15} or (R_{I}) multiplied by γ^2.

$$R_3 = - \frac{a \mathrm{d} R_1}{2 \mathrm{d} a} - R_1 \qquad R_1 = \frac{a}{8 a_{\prime}^2} (b_{3,0} + b_{3,2})$$

$$R_3 = - \frac{a}{16 a_{\prime}^2} (b_{3,0} + b_{3,2} + \frac{3 a^2}{16 a_{\prime}^3} \left\{ \frac{a}{a_{\prime}} b_{5,0} - b_{5,1} + \frac{a}{a_{\prime}} b_{5,2} - \frac{1}{2} b_{5,1} - \frac{1}{2} b_{5,3} \right\}$$

$$- \frac{a}{8 a_{\prime}^2} (b_{3,0} + b_{3,2}$$

$$= - \frac{3 a}{16 a_{\prime}^2} \left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2} b_{5,0} - 2 \frac{a}{a_{\prime}} b_{5,1} \right) - \frac{a}{16 a_{\prime}^2} b_{3,2} + \frac{3 a^3}{16 a_{\prime}^4} b_{5,0} - \frac{3 a^2}{16 a_{\prime}^3} b_{5,1}$$

$$+ \frac{3 a^3}{16 a_{\prime}^4} b_{5,2} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,1} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,3} - \frac{3 a^2}{32 a_{\prime}^3} b_{5,1} + \frac{3 a^2}{32 a_{\prime}^3} b_{5,3}$$

$$= - \frac{a}{16 a_{\prime}^2} b_{3,2} - \frac{3 a}{16 a_{\prime}^2} b_{5,0} + \frac{3 a^3}{16 a_{\prime}^4} b_{5,2}.$$

If the term in R_{15} multiplied by γ^2 be called R''_{15},

$$R''_{15} = - \frac{a_, \mathrm{d}\, R_3}{2\, \mathrm{d}\, a_,} - R_3$$

$$= - \frac{a}{16\, a_,^2} b_{3,2} - \frac{3\, a}{16\, a_,^2} b_{5,0} + \frac{3\, a^3}{8\, a_,^4} b_{5,2} + \frac{15}{32} \left\{ \frac{a^2}{a_,^3} \left(\frac{a}{a_,} b_{7,0} - b_{7,1} \right) \right.$$

$$\left. - \frac{a^4}{a_,^5} \left(\frac{a}{a_,} b_{7,2} - \frac{1}{2} b_{7,1} - \frac{1}{2} b_{7,3} \right) \right\} + \frac{a}{16\, a_,^2} b_{3,2} + \frac{3\, a}{16\, a_,^2} b_{5,0} - \frac{3\, a^3}{16\, a_,^4} b_{5,2}$$

$$= \frac{3\, a^3}{16\, a_,^4} b_{5,2} + \frac{15}{32} \left\{ - \frac{a^2}{a_,^3} \left(\frac{a^2 + a_,^2}{a_,^2} b_{7,1} - \frac{a}{a_,} b_{7,0} - \frac{a}{a_,} b_{7,2} \right) \right.$$

$$\left. - \frac{a^3}{a_,^4} \left(\frac{a^2 + a_,^2}{a_,^2} b_{7,2} - \frac{a}{a_,} b_{7,1} - \frac{a}{a_,} b_{7,3} \right) + \frac{a^4}{2\, a_,^5} b_{7,1} - \frac{a^4}{2\, a_,^5} b_{7,3} \right) \right\}$$

$$= \frac{3\, a^3}{16\, a_,^4} b_{5,2} - \frac{15\, a^2}{32\, a_,^3} b_{5,1} - \frac{15\, a^3}{32\, a_,^4} b_{5,2} + \frac{3\, a^3}{16\, a_,^4} b_{5,2}$$

$$= - \frac{15\, a^2}{32\, a_,^3} b_{5,1} - \frac{3\, a^3}{32\, a_,^2} b_{5,2}.$$

Therefore *R* contains the term

$$m_, \left\{ - \frac{15\, a_,^2}{32\, a_,^3} b_{5,1} - \frac{3\, a^3}{32\, a_,^4} b_{5,2} \right\} e\, e_, (\gamma^2 + \gamma_,^2) \cos (\tau - \xi + \xi_,).$$

Calculation of R_{III}, *or the Coefficient of* $\cos (\tau + \xi + \xi_, - 2\, \eta)$, *in the Development of R.*

Distinguishing at present the argument $\tau + \xi_, - 2\, \eta$ by the index 7, and the argument $\tau - 2\, \eta$ by the index 1,

$$R_7 = - \frac{a_, \mathrm{d}\, R_1}{2\, \mathrm{d}\, a_,} - R_1 \qquad\qquad R_1 = - \frac{a}{8\, a_,^2} b_{3,0}$$

$$R_7 = - \frac{a}{8\, a_,^2} b_{3,0} + \frac{3\, a^2}{16\, a_,^3} \left(\frac{a}{a_,} b_{5,0} - b_{5,1} \right) + \frac{a}{8\, a_,^2} b_{3,0} = \frac{3\, a^3}{16\, a_,^4} b_{5,0} - \frac{3\, a^2}{16\, a_,^3} b_{5,1}$$

$$R_{\mathrm{III}} = - \frac{a\, \mathrm{d}\, R_7}{2\, \mathrm{d}\, a} - R_7$$

$$= - \frac{9\, a^3}{32\, a_,^4} b_{5,0} + \frac{2\, a^2}{16\, a_,^3} b_{5,1} + \frac{15}{32} \left\{ \frac{a^4}{a_,^5} \left(\frac{a}{a_,} b_{7,0} - b_{7,1} \right) \right.$$

$$\left. - \frac{a^3}{a_,^4} \left(\frac{a}{a_,} b_{7,1} - \frac{1}{2} b_{0,0} + \frac{1}{2} b_{7,2} \right) \right\} - \frac{3\, a^3}{16\, a_,^4} b_{5,0} + \frac{3\, a^2}{16\, a_,^3} b_{5,1}$$

$$= - \frac{15\, a^3}{32\, a_,^4} b_{5,0} + \frac{15}{32} \left\{ \frac{a^3}{a_,^4} \left(\frac{a^2 + a_,^2}{a_,^2} b_{7,0} + 2 \frac{a}{a_,} b_{7,1} \right) - \frac{a^3}{2\, a_,^4} b_{7,0} + \frac{a^3}{2\, a_,^4} b_{7,2} \right\} + \frac{3\, a^2}{8\, a_,^3} b_{5,1}$$

$$- \frac{15\, a^3}{32\, a_,^4} b_{5,0} + \frac{15\, a^3}{32\, a_,^4} b_{5,0} + \frac{3\, a^2}{32\, a_,^3} b_{5,1} + \frac{3\, a^2}{8\, a_,^3} b_{5,1} = \frac{9\, a_,^2}{32\, a_,^3} b_{5,1}.$$

Hence *R* contains the term $\frac{9\, m_, a^2}{32\, a_,^3} b_{5,1}\, e\, e_, \gamma^2 \cos (\tau + \xi + \xi_, - 2\, \eta)$.

Calculation of R_{IV}, or the Coefficient of cos $(\tau - \xi - \xi_{,} + 2\eta_{,})$, *in the Development of R.*

Distinguishing at present the argument $\tau - \xi + 2\eta_{,}$ by the index 3, and $\tau + 2\eta_{,}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R}{2\,\mathrm{d}\,a} - R_1 \qquad\qquad R_1 = -\frac{a}{8\,a_{,}^2}\,b_{3,0}$$

$$R_3 = \frac{a}{16\,a_{,}^2}\,b_{3,0} - \frac{3\,a^2}{16\,a_{,}^3}\left(\frac{a}{a_{,}}\,b_{5,0} - b_{5,1}\right) + \frac{a}{8\,a_{,}^2}\,b_{3,0}$$

$$= \frac{3\,a}{16\,a_{,}^2}\,b_{3,0} - \frac{3\,a^3}{16\,a_{,}^4}\,b_{5,0} + \frac{3\,a^2}{16\,a_{,}^3}\,b_{5,1}$$

$$R_{IV} = -\frac{a_{,}\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a_{,}} - R_3$$

$$= \frac{3\,a}{16\,a_{,}^2}\,b_{3,0} - \frac{3\,a^3}{8\,a_{,}^4}\,b_{5,0} + \frac{9\,a^2}{32\,a_{,}^3}\,b_{5,1} - \frac{9\,a^2}{32\,a_{,}^3}\left(\frac{a}{a_{,}}\,b_{5,0} - b_{5,1}\right)$$

$$+ \frac{15\,a^3}{32\,a_{,}^4}\,b_{5,0} - \frac{3\,a^2}{32\,a_{,}^3}\,b_{5,1} - \frac{3\,a}{16\,a_{,}^2}\,b_{3,0} + \frac{3\,a^3}{16\,a_{,}^4}\,b_{5,0} - \frac{3\,a^2}{16\,a_{,}^3}\,b_{5,1}$$

$$= \frac{9\,a^2}{32\,a_{,}^3}\,b_{5,1}.$$

Hence R contains the term $\frac{9\,m_{,}\,a^2}{32\,a_{,}^3}\,b_{5,1}\,e\,e_{,}\,\gamma_{,}^2\cos(\tau - \xi - \xi_{,} + 2\,\eta_{,})$.

Calculation of R_{V}, or the Coefficient of cos $(\xi - \xi_{,} - \eta + \eta_{,})$, *in the Development of R.*

Distinguishing the argument $\eta - \eta_{,} - \xi$ by the index 3, and $\eta - \eta_{,}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R_{,}}{2\,\mathrm{d}\,a_{,}} - R_1 \qquad\qquad R_1 = -\frac{a}{4\,a_{,}^2}\,b_{3,0}$$

$$R_{V} = -\frac{a_{,}\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a_{,}} - R_3 = \frac{9\,a^2}{16\,a_{,}^3}\,b_{5,1}$$

Hence R contains the term $\frac{9\,m_{,}\,a^2}{16\,a_{,}^3}\,b_{5,1}\,e\,e_{,}\,\gamma\,\gamma_{,}\cos(\xi - \xi_{,} - \eta + \eta_{,})$.

Calculation of R_{VI}, or the Coefficient of cos $(\xi + \xi_{,} - \eta - \eta_{,})$, *in the Development of R.*

Distinguishing the argument $\eta + \eta_{,} - \xi$ by the index 3, and $\eta + \eta_{,}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - R_1 \qquad\qquad R_1 = \frac{a}{4\,a_{,}^2}\,b_{3,0}$$

$$R_{VI} = -\frac{a_{,}\mathrm{d}\,R_3}{2\,\mathrm{d}\,a_{,}} - R_3 = -\frac{9\,a^2}{16\,a_{,}^3}\,b_{5,1}.$$

Hence R contains the term $-\frac{9\,m_{,}\,a^2}{16\,a_{,}^3}\,b_{5,1}\cos(\xi + \xi_{,} - \eta - \eta_{,})$.

Calculation of R_{VII}, or the Coefficient of cos $(2\,\tau + 2\,\xi_{,} - 2\,\eta)$, *in the Development of R.*

Distinguishing the argument $2\,\tau - 2\,\eta + \xi_{,}$ by the index 7, and $2\,\tau - 2\,\eta$ by the index 1,

$$R_7 = -\frac{a_{,} \mathrm{d} R_1}{2 \mathrm{d} a_{,}} - 2 R_1 \qquad R_1 = -\frac{a}{8 a_{,}^2} b_{3\,1}$$

$$= -\frac{a}{8 a_{,}^2} b_{3\,1} + \frac{3 a^2}{16 a_{,}^3} \left(\frac{a}{a_{,}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) + \frac{a}{4 a_{,}^2} b_{3,1}$$

$$= \frac{a}{8 a_{,}^2} b_{3,1} + \frac{3 a^3}{16 a_{,}^4} b_{5,1} - \frac{3 a^2}{32 a_{,}^3} b_{5,0} - \frac{3 a^2}{32 a_{,}^3} b_{5,2}$$

$$= \frac{3 a^2}{16 a_{,}^4} b_{5,0} - \frac{3 a^2}{16 a_{,}^4} b_{5,2} + \frac{3 a^3}{16 a_{,}^4} b_{5,1} - \frac{3 a^2}{32 a_{,}^3} b_{5,0} - \frac{3 a^2}{32 a_{,}^3} b_{5,2}$$

$$= -\frac{3 a^2}{32 a_{,}^3} b_{5,0} + \frac{3 a^3}{16 a_{,}^4} b_{5,1} - \frac{9 a^2}{32 a_{,}^3} b_{5,2}$$

$$= \frac{3 a^2}{16 a_{,}^3} \text{ in the lunar theory.}$$

$$2 R_{\text{VII}} = -\frac{a_{,} \mathrm{d} R_7}{2 \mathrm{d} a} - 2 R_7 - \frac{3}{4} \frac{a_{,} \mathrm{d} R_1}{\mathrm{d} a_{,}} - \frac{5}{2} R_1$$

$$-\frac{R_7}{2} - \frac{3}{4} R_1 = -\frac{3 a^2}{64 a_{,}^3} b_{5,0} - \frac{3 a^3}{32 a_{,}^4} b_{5,1} + \frac{9 a^2}{64 a_{,}^3} b_{5,2} + \frac{9 a^2}{64 a_{,}^3} b_{5,0} - \frac{9 a^2}{64 a_{,}^3} b_{5,2}$$

$$= \frac{3 a^2}{32 a_{,}^3} b_{5,0} - \frac{3 a^3}{32 a_{,}^4} b_{5,1}$$

$$-2 R_7 - \frac{5}{2} R_1 = -\frac{3 a^2}{16 a_{,}^3} b_{5,0} - \frac{3 a^3}{8 a_{,}^4} b_{5,1} + \frac{9 a^2}{16 a_{,}^3} b_{5,2} + \frac{15 a^2}{32 a_{,}^3} b_{5,0} - \frac{15 a^2}{32 a_{,}^3} b_{5,2}$$

$$= \frac{9 a^2}{32 a_{,}^3} b_{5,0} - \frac{3 a^3}{16 a_{,}^4} b_{5,1} + \frac{3 a^2}{32 a_{,}^3} b_{5,2}$$

$$2 R_{\text{VII}} = -\frac{9 a^2}{32 a_{,}^3} b_{5,0} + \frac{3 a^3}{8 a_{,}^4} b_{5,1} + \frac{15}{32} \left\{ \frac{a^3}{a_{,}^4} \left(\frac{a}{a_{,}} b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{,}^5} \left(\frac{a}{a_{,}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right) \right\}$$

$$+ \frac{9 a^2}{32 a_{,}^3} b_{5\,0} - \frac{3 a^3}{8 a_{,}^4} b_{5,1} + \frac{3 a^2}{32 a_{,}^3} b_{5,2}$$

$$= \frac{3 a^2}{32 a_{,}^3} b_{5,2} + \frac{15}{32} \left\{ -\frac{a^3}{a_{,}^4} \left(\frac{a^2 + a_{,}^2}{a_{,}^2} b_{7,1} - \frac{a}{a_{,}} b_{7,0} - \frac{a}{a_{,}} b_{7,2}\right) + \frac{a^4}{2 a_{,}^5} b_{7,0} - \frac{a^4}{2 a_{,}^5} b_{7,2} \right\}$$

$$= \frac{3 a^2}{32 a_{,}^3} b_{5,2} - \frac{15 a^3}{32 a_{,}^4} b_{5,1} + \frac{3 a^3}{32 a_{,}^4} b_{5,1}$$

$$= -\frac{3 a^3}{8 a_{,}^4} b_{5,1} + \frac{3 a^2}{32 a_{,}^3} b_{5,2}.$$

Hence R contains the term $m_{,} \left\{ -\frac{3 a^3}{16 a_{,}^4} b_{5,1} + \frac{3 a^2}{64 a_{,}^3} b_{5,2} \right\} e_{,}^2 \gamma^2 \cos (2 \tau + 2 \xi_{,} - 2 \eta)$.

Calculation of R_{VII}, *or the Coefficient of* cos $(2 \xi - 2 \eta)$, *in the Development of R.*

Distinguishing the argument $\xi - 2 \eta$ by the index 65, and the argument 2η by the index 62,

$$R_{65} = -\frac{a \mathrm{d} R_{62}}{2 \mathrm{d} a} - 2 R_{62} \qquad R_{62} = -\frac{a}{8 a_{,}^2} b_{3,1}$$

$$R_{65} = \frac{a}{16\,a_{\prime}^{2}}\,b_{3,1} - \frac{3\,a^2}{16\,a_{\prime}^{3}}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{a}{4\,a_{\prime}^{2}}\,b_{3,1}$$

$$= \frac{3\,a^2}{32\,a_{\prime}^{3}}\,b_{5,0} - \frac{3\,a^2}{32\,a_{\prime}^{3}}\,b_{5,2} - \frac{3\,a^3}{16\,a_{\prime}^{4}}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^{3}}\,b_{5,0} + \frac{3\,a^2}{32\,a_{\prime}^{3}}\,b_{5,2} + \frac{a}{4\,a_{\prime}^{2}}\,b_{3,1}$$

$$= \frac{a}{4\,a_{\prime}^{2}}\,b_{3,1} + \frac{3\,a^2}{16\,a_{\prime}^{3}}\,b_{5,0} - \frac{3\,a^3}{16\,a_{\prime}^{4}}\,b_{5,1}$$

$$= \frac{9\,a^2}{8\,a_{\prime}^{3}} \text{ in the lunar theory.}$$

$$2\,R_{\text{VIII}} = -\frac{a\,\mathrm{d}\,R_{65}}{2\,\mathrm{d}\,a} - 2\,R_{65} - \frac{3\,a\,\mathrm{d}\,R_{62}}{4\,\mathrm{d}\,a} - \frac{5}{2}\,R_{62}$$

$$= -\frac{3\,a^2}{16\,a_{\prime}^{3}}\,b_{5,0} + \frac{9\,a^3}{32\,a_{\prime}^{4}}\,b_{5,1} + \frac{9\,a^2}{32\,a_{\prime}^{3}}\,b_{5,0} - \frac{9\,a^3}{32\,a_{\prime}^{4}}\,b_{5,1}$$

$$- \frac{a}{8\,a_{\prime}^{2}}\,b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^{3}}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$+ \frac{15}{32}\left\{\frac{a^3}{a_{\prime}^{4}}\left(\frac{a}{a_{\prime}}\,b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^{5}}\left(\frac{a}{a_{\prime}}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)\right\}$$

$$- \frac{3\,a^2}{8\,a_{\prime}^{3}}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^{4}}\,b_{5,1} - \frac{a}{2\,a_{\prime}^{2}}\,b_{3,1} + \frac{5\,a}{16\,a_{\prime}^{2}}\,b_{3,1}$$

$$= -\frac{15\,a^2}{32\,a_{\prime}^{3}}\,b_{5,0} - \frac{15\,a^2}{32\,a_{\prime}^{3}}\,b_{5,0} + \frac{15\,a^2}{32\,a_{\prime}^{3}}\,b_{5,2} + \frac{3\,a^3}{4\,a_{\prime}^{4}}\,b_{5,1}$$

$$+ \frac{15}{32}\left\{-\frac{a^3}{a_{\prime}^{4}}\left(\frac{a^2 + a_{\prime}^{2}}{a_{\prime}^{2}}\,b_{7,1} - \frac{a}{a_{\prime}}\,b_{7,0} - \frac{a}{a_{\prime}}\,b_{7,2}\right) + \frac{a^4}{2\,a_{\prime}^{5}}\,b_{7,0} - \frac{a^4}{2\,a_{\prime}^{5}}\,b_{7,2}\right\}$$

$$= -\frac{15\,a^2}{16\,a_{\prime}^{3}}\,b_{5,0} + \frac{9\,a^2}{32\,a_{\prime}^{3}}\,b_{5,2} + \frac{3\,a^3}{4\,a_{\prime}^{4}}\,b_{5,1} - \frac{15\,a^3}{32\,a_{\prime}^{4}}\,b_{5,1} + \frac{3\,a^3}{32\,a_{\prime}^{4}}\,b_{5,1}$$

$$= -\frac{15\,a^2}{16\,a_{\prime}^{3}}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^{4}}\,b_{5,1} + \frac{9\,a^2}{32\,a_{\prime}^{3}}\,b_{5,2} = -\frac{3\,a}{8\,a_{\prime}^{2}}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^{3}}\,b_{5,2}$$

$$= -\,*\,\frac{15 a^2}{8\,a_{\prime}^{3}} \text{ in the lunar theory.}$$

Hence R contains the term $\left\{-\frac{3\,a}{16\,a_{\prime}^{2}}\,b_{5,1} + \frac{3\,a^2}{64\,a_{\prime}^{3}}\,b_{5,2}\right\} e^2\,\gamma^2 \cos\left(2\,\xi - 2\,\eta\right)$.

Calculation of R_{IX}, *or the Coefficient of* $\cos\left(2\,\tau - 2\,\xi + 2\,\eta_{\prime}\right)$, *in the Development of R.*

Distinguishing the argument $2\,\tau - \xi + 2\,\eta_{\prime}$ by the index 3, and $2\,\tau + 2\,\eta_{\prime}$ by the index 1,

$$R_3 = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}_2} - R_1 \qquad\qquad R_1 = -\frac{a}{8\,a_{\prime 2}}\,b_{3,1}$$

$$R_3 = \frac{a}{16\,a_{\prime}^{2}}\,b_{3,1} - \frac{3\,a^2}{16\,a_{\prime}^{3}}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{a}{4\,a_{\prime}^{2}}\,b_{3,1}$$

* R_{VIII} (or R_{77}) $= -\frac{15\,a^2}{16 a_{\prime}^{3}}$, this agrees with the result I arrived at formerly, since confirmed by M. POISSON.

$$= \frac{3\,a^2}{32\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,2} - \frac{3\,a^3}{16\,a_i^4}\,b_{5,1} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,2} + \frac{a}{4\,a_i^2}\,b_{3,1}$$

$$= \frac{a}{4\,a_i^2}\,b_{3,1} + \frac{3\,a^3}{32\,a_i^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}$$

$$2\,R_{\text{IX}} = -\frac{a\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a} - 2\,R_3 - \frac{3}{4}\frac{a\,\mathrm{d}\,R_1}{\mathrm{d}\,a} - \frac{5}{2}\,R_1$$

$$-\frac{1}{2}\,R_3 - \frac{3}{4}\,R_1 = -\frac{a}{3\,a_i^2}\,b_{3,1} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{32\,a_i^4}\,b_{5,1} + \frac{3\,a}{32\,a_i^2}\,b_{3,1}$$

$$= -\frac{a}{32\,a_i^2}\,b_{3,1} - \frac{3\,a^2}{32\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{32\,a_i^4}\,b_{5,1}$$

$$-2\,R_3 - \frac{5}{2}\,R_1 = -\frac{3\,a^2}{8\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_i^4}\,b_{5,1} - \frac{a}{8\,a_i^2}\,b_{3,1} + \frac{5\,a}{16\,a_i^2}\,b_{3,1}$$

$$= -\frac{3\,a^2}{8\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_i^4}\,b_{5,1} - \frac{3\,a}{16\,a_i^2}\,b_{3,1}$$

$$2\,R_{\text{IX}} = -\frac{a}{32\,a_i^2}\,b_{3,1} + \frac{3\,a^2}{16\,a_i^3}\,b_{5,3} + \frac{9\,a^3}{32\,a_i^4}\,b_{5,1} + \frac{3\,a^2}{32\,a_i^3}\left(\frac{a}{a_i}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$+\frac{15}{32}\left\{\frac{a^3}{a_i^4}\left(\frac{a}{a_i}\,b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_i^5}\left(\frac{a}{a_i}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)\right\}$$

$$-\frac{3\,a^2}{8\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_i^4}\,b_{5,1} - \frac{3\,a}{16\,a_i^2}\,b_{3,1}$$

$$= -\frac{7\,a}{32\,a_i^2}\,b_{3,1} - \frac{39\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{4\,a_i^4}\,b_{5,1} - \frac{3\,a}{64\,a_i^2}\,b_{5,2}$$

$$+\frac{15}{32}\left\{-\frac{a^3}{a_i^4}\left(\frac{a^2+a_i^2}{a_i^2}\,b_{7,1} - \frac{a}{a_i}\,b_{7,0} - \frac{a}{a_i}\,b_{7,2}\right) + \frac{a^4}{2\,a_i^5}\,b_{7,0} - \frac{a^4}{2\,a_i^5}\,b_{7,2}\right\}$$

$$= -\frac{21\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{21\,a^2}{64\,a_i^3}\,b_{5,2} - \frac{39\,a^2}{64\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{4\,a_i^4}\,b_{5,1} - \frac{3\,a}{64\,a_i^2}\,b_{5,2} - \frac{15\,a^3}{32\,a_i^4}\,b_{5,1} + \frac{3\,a^3}{32\,a_i^4}\,b_{5,1}$$

$$= -\frac{3\,a}{8\,a_i^2}\,b_{5,1} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}.$$

Hence R contains the term $\left\{-\frac{3\,a}{16\,a_i^2}\,b_{5,1} + \frac{3\,a^2}{32\,a_i^3}\,b_{5,2}\right\} e^2\,\gamma_i^2 \cos\,(2\,\tau - 2\,\xi + 2\,\eta_i)$.

Calculation of R_{X}, *or the Coefficient of* $\cos\,(2\,\xi_i - 2\,\eta_i)$, *in the Development of* R.

Distinguishing the argument $2\,\eta_i - \xi_i$ by the index 65, and the argument $2\,\eta_i$ by the index 62,

$$R_{65} = -\frac{a_i\mathrm{d}\,R_{62}}{2\,\mathrm{d}\,a_i} - 2\,R_{62} \qquad R_{62} = -\frac{a}{8\,a_i^2}\,b_{3,1}$$

$$R_{65} = -\frac{a}{8\,a_i^2}\,b_{3,1} + \frac{3\,a^2}{15\,a_i^3}\left(\frac{a}{a_i}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{a}{4\,a_i^2}\,b_{3,1}$$

$$= \frac{3\,a}{16\,a_i^2}\,b_{3,1} - \frac{3\,a^2}{16\,a_i^3}\,b_{5,0} + \frac{3\,a^3}{16\,a_i^4}\,b_{5,1}$$

$$2\,R_{\rm x} = -\frac{a_{\prime}\mathrm{d}R_{65}}{2\,\mathrm{d}\,a_{\prime}} - 2\,R_{65} - \frac{3\,a_{\prime}\,\mathrm{d}\,R_{62}}{4\,\mathrm{d}\,a_{\prime}} - \frac{5}{2}\,R_{62}$$

$$= \frac{3\,a}{16\,a_{\prime}^2}\,b_{3,1} - \frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{9\,a^2}{32\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$-{*}\,\frac{15\,a^3}{32\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^3}{32\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a}{8\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a}{16\,a_{\prime}^2}\,b_{3,1}$$

$$+ \frac{9\,a^2}{32\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right) + \frac{5\,a}{16\,a_{\prime}^2}\,b_{3,1}$$

$$= \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{a}{16\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1}$$

$$= -\frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}.$$

Hence R contains the term $\left\{-\frac{3\,a^3}{16\,a_{\prime}^4}\,b_{5\,1} + \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2}\right\} e_{\prime}^2\,\gamma_{\prime}^2 \cos(2\,\xi_{\prime} - 2\,\eta_{\prime})$.

Calculation of the Term in $R_{\rm XI}$ *multiplied by* e^2.

Distinguishing the argument $\tau - \eta + \eta_{\prime}$ by the index 1, $\tau - \eta + \eta_{\prime} - \xi$ by the index 3, and $\tau - \eta + \eta_{\prime} + \xi$ by the index 4,

$$R_3 = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} \qquad R_4 = -\frac{a\mathrm{d}R_1}{2\,\mathrm{d}\,a} = R_3 \qquad R_1 = -\frac{a}{4\,a_{\prime}^2}\,b_{3,1}$$

$$R_3 = \frac{a}{8\,a_{\prime}^2}\,b_{3,1} - \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$= \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5\,0} - \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,2} - \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,2}$$

$$= \frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1}$$

$$2\,R_{\rm XI} = \frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R_4}{2\,\mathrm{d}\,a} = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - \frac{a\,\mathrm{d}\,R_3}{\mathrm{d}\,a}$$

$$= -\frac{3\,a^2}{4\,a_{\prime}^3}\,b_{5,0} + \frac{9\,a^3}{8\,a_{\prime}^4}\,b_{5,1} + \frac{15}{8}\left\{\frac{a}{a_{\prime}^4}\left(\frac{a}{a_{\prime}}\,b_{\prime,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}}\,b_{7,1} - \frac{1}{2}\,b_{7,0} - \frac{1}{2}\,b_{7,2}\right)\right\}$$

$$- \frac{a}{8\,a_{\prime}^2}\,b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,1} - \frac{1}{2}\,b_{5,0} - \frac{1}{2}\,b_{5,2}\right)$$

$$= -\frac{3\,a^2}{4\,a_{\prime}^3}b_{5,0} + \frac{9\,a^3}{8\,a_{\prime}^4}b_{5,1} + \frac{15}{8}\left\{-\frac{a^3}{a_{\prime}^4}\left(\frac{a^2+a_{\prime}^2}{a_{\prime}^2}\,b_{7,1} - \frac{a}{a_{\prime}}b_{7,0} - \frac{a}{a_{\prime}}b_{7,2}\right) + \frac{a^4}{2\,a_{\prime}^5}\,b_{7,0} - \frac{a^4}{2\,a_{\prime}^5}b_{7,2}\right\}$$

$$- \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5\,0} + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,2} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,2}$$

$$= -\frac{3\,a^2}{4\,a_{\prime}^3}\,b_{5,0} + \frac{9\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{15\,a^3}{8\,a_{\prime}^4}\,b_{5,1} + \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0}$$

* For certain reductions which occur here, see the calculation of $R_{\rm VIII}$.

$$+ \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2}$$

$$= - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0}.$$

Therefore R contains the term $- \frac{9\,a^2}{16\,a_{\prime}^3} b_{5,0}\, e^2 \gamma \gamma_{\prime} \cos(\tau - \eta + \eta_{\prime})$.

Calculation of the Term in R_{XI} multiplied by e^2.

Distinguishing the argument $\tau - \eta + \xi_{\prime}$ by the index 1, the argument $\tau - \eta + \eta_{\prime} - \xi_{\prime}$ by the index 6, and $\tau - \eta + \eta_{\prime} + \xi_{\prime}$ by the index 7,

$$R_6 = - \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} \qquad R_7 = - \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} = R_6 \qquad R_1 = - \frac{a}{4\,a_{\prime}^2} b_{3,1}$$

$$R_6 = - \frac{a}{4\,a_{\prime}^2} b_{3,1} + \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$= - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} - \frac{a}{8\,a_{\prime}^2} b_{3,1}$$

$$= - \frac{a}{8\,a_{\prime}^2} b_{3,1} - \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1}$$

$$2\,R_{\text{XI}} = \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\,\mathrm{d}\,R_6}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\,\mathrm{d}\,R_7}{2\,\mathrm{d}\,a_{\prime}} = \frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} - \frac{a_{\prime}\,\mathrm{d}\,R_6}{\mathrm{d}\,a_{\prime}}$$

$$2\,R_{\text{XI}} = - \frac{9\,a^2}{8\,a_{\prime}^2} b_{5,0} + \frac{3\,a^3}{2\,a_{\prime}^4} b_{5,1} - \frac{a}{4\,a_{\prime}^2} b_{3,1}$$

$$+ \frac{15}{8}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}} b_{7,0} - b_{7,1}\right) - \frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}} b_{7,1} - \frac{1}{2} b_{7,0} - \frac{1}{2} b_{7,2}\right)\right\}$$

$$+ \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right) + \frac{a}{4\,a_{\prime}^2} b_{3,1} - \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}} b_{5,1} - \frac{1}{2} b_{5,0} - \frac{1}{2} b_{5,2}\right)$$

$$= - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^3}{2\,a_{\prime}^4} b_{5,1} - \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,0} + \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,2} - \frac{15\,a^3}{8\,a_{\prime}^4} b_{5,1} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} + \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1}$$

$$- \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2} + \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,0} - \frac{3\,a^2}{8\,a_{\prime}^3} b_{5,2} - \frac{3\,a^3}{8\,a_{\prime}^4} b_{5,1} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,0} + \frac{3\,a^2}{16\,a_{\prime}^3} b_{5,2}$$

$$= - \frac{9\,a^2}{8\,a_{\prime}^3} b_{5,0}.$$

Therefore R contains the term $- \frac{9\,a^2}{16\,a_{\prime}^3} b_{5,0}\, e_{\prime}^2 \gamma \gamma_{\prime} \cos(\tau - \eta + \eta_{\prime})$.

Calculation of R_{XII} or the Coefficient of $\cos(\tau - 2\,\xi + \eta + \eta_{\prime})$ *in the Development of R.*

Distinguishing the argument $\tau + \eta + \eta_{\prime} - \xi$ by the index 3, and $\tau + \eta + \eta_{\prime}$ by the index 1,

$$R_3 = - \frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - 2\,R_1 \qquad R_1 = \frac{a}{4\,a_{\prime}^2} b_{3,1}$$

$$2\,R_{\text{XII}} = -\frac{a\,\mathrm{d}\,R_3}{2\,\mathrm{d}\,a} - 2\,R_3 - \frac{3}{4}\frac{a\,\mathrm{d}\,R_1}{\mathrm{d}\,a} - \frac{5}{2}R_1.$$

It is evident from the calculation of R_{VIII} that R contains the term

$$\left\{\frac{3\,a}{8\,a_{\prime}^2}\,b_{5,1} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}\right\} e^2\,\gamma\,\gamma_{\prime}\cos\left(\tau - 2\,\xi + \eta + \eta_{\prime}\right).$$

It is evident similarly, and from the calculation of R_{X}, that R contains the term

$$\left\{\frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,1} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}\right\} e_{\prime}^2\,\gamma\,\gamma_{\prime}\cos\left(\tau + 2\,\xi_{\prime} - \eta + \eta_{\prime}\right).$$

Calculation of R_{XIV} *or the Coefficient of* $\cos\left(2\,\tau - \xi + \xi_{\prime} - \eta + \eta_{\prime}\right)$.

Distinguishing the argument $2\,\tau - \eta + \eta_{\prime} + \xi_{\prime}$ by the index 7, and the argument $2\,\tau - \eta + \eta_{\prime}$ by the index 1,

$$R_7 = -\frac{a_{\prime}\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a_{\prime}} - R_1 \qquad\qquad R_1 = -\frac{a}{4\,a_{\prime}^2}\,b_{3,2}$$

$$R_7 = \frac{a}{4\,a_{\prime}^2}\,b_{3,2} + \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,2} - \frac{1}{2}\,b_{5,1} - \frac{1}{2}\,b_{5,3}\right) + \frac{a}{4\,a_{\prime}^2}\,b_{3,2}$$

$$= \frac{a}{8\,a_{\prime}^2}\,b_{3,2} - \frac{3\,a^3}{16\,a_{\prime}^4}\,b_{5,2} + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,2}$$

$$R_{\text{XIV}} = -\frac{a\,\mathrm{d}\,R_7}{2\,\mathrm{d}\,a} - R_7$$

$$= -\frac{a}{8\,a_{\prime}^4}\,b_{3,2} - \frac{9\,a^3}{16\,a_{\prime}^4}\,b_{5,2} + \frac{3\,a^2}{8\,a_{\prime}^3}\,b_{5,3} - \frac{3\,a^2}{8\,a_{\prime}^3}\left(\frac{a}{a_{\prime}}\,b_{5,2} - \frac{1}{2}\,b_{5,1} - \frac{1}{2}\,b_{5,3}\right)$$

$$+ \frac{15}{16}\left\{\frac{a^4}{a_{\prime}^5}\left(\frac{a}{a_{\prime}}\,b_{7,2} - \frac{1}{2}\,b_{7,1} - \frac{1}{2}\,b_{7,3}\right) - \frac{a^3}{a_{\prime}^4}\left(\frac{a}{a_{\prime}}\,b_{7,3} - \frac{1}{2}\,b_{7,2} - \frac{1}{2}\,b_{7,4}\right)\right\}$$

$$+ \frac{a}{4\,a_{\prime}^2}\,b_{3,2} - \frac{3\,a^3}{8\,a_{\prime}^4}\,b_{5,2} + \frac{3\,a^2}{8\,a_{\prime}^3}\,b_{3,5}$$

$$= -\frac{21\,a^3}{16\,a_{\prime}^4}\,b_{5,2} + \frac{15\,a^2}{8\,a_{\prime}^3}\,b_{5,3} + \frac{3\,a}{8\,a_{\prime}^2}\,b_{3,2}$$

$$+ \frac{15}{16}\left\{\frac{a^3}{a_{\prime}^4}\left(\frac{a^2 + a_{\prime}^2}{a_{\prime}^2}b_{7,2} - \frac{a}{a_{\prime}}b_{7,1} - \frac{a}{a_{\prime}}b_{7,3}\right) - \frac{a^3}{2\,a_{\prime}^2}b_{7,2} + \frac{a^3}{2\,a^{\prime 4}}b_{7,4} + \frac{a^4}{2\,a_{\prime}^5}b_{7,1} - \frac{a^4}{2\,a_{\prime}^5}b_{7,3}\right\}$$

$$= -\frac{21\,a^3}{16\,a_{\prime}^4}b_{5,2} + \frac{15\,a^2}{8\,a_{\prime}^3}b_{5,3} + \frac{9\,a^2}{32\,a_{\prime}^3}b_{5,1} - \frac{9\,a^2}{32\,a_{\prime}^3}b_{5,3} + \frac{15\,a^3}{16\,a_{\prime}^4}b_{5,2} - \frac{9\,a^2}{16\,a_{\prime}^3}b_{5,3} + \frac{3\,a^3}{8\,a_{\prime}^4}b_{5,2}$$

$$= \frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,1} + \frac{33\,a^2}{32\,a_{\prime}^3}\,b_{5,3}.$$

Therefore R contains the term

$$\left\{\frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,1} + \frac{33\,a^2}{32\,a_{\prime}^3}\,b_{5,3}\right\} e\,e_{\prime}\,\gamma\,\gamma_{\prime}\cos\left(2\,\tau - \xi + \xi_{\prime} - \eta + \eta_{\prime}\right)$$

$R =$ *terms independent of the quantities b

$$- \frac{m}{a} \left\{ \frac{1}{2} b_{1,0} + b_{1,1} \cos \tau + b_{1,2} \cos 2\tau +, \&c. \right\}$$

$$- \frac{m_{\prime} a}{a_{\prime}^2} \left\{ \frac{1}{2} b_{3,0} + b_{3,1} \cos \tau + b_{3,2} \cos 2\tau +, \&c. \right\}$$

$$\left\{ - \frac{\gamma^2}{4} - \frac{\gamma_{\prime}^2}{4} + \frac{3}{16}\gamma^4 + \frac{\gamma^2\gamma_{\prime}^2}{16} + \frac{3}{16}\gamma_{\prime}^4 \right\} \cos \tau + \frac{\gamma^2}{4}\left(1 - \frac{\gamma_{\prime}^2}{4}\right)(\cos \tau - 2\eta)$$

$$+ \frac{\gamma_{\prime}^2}{4}\left(1 - \frac{\gamma^2}{4}\right) \cos(\tau + 2\eta_{\prime}) + \frac{\gamma^2\gamma_{\prime}^2}{16} \cos(\tau - 2\eta + 2\eta_{\prime})$$

$$+ \frac{\gamma\gamma_{\prime}}{2}\left(1 - \frac{\gamma^3}{4} - \frac{\gamma_{\prime}^2}{4}\right) \cos(\eta - \eta_{\prime}) - \frac{\gamma\gamma_{\prime}}{2}\left(1 - \frac{\gamma^2}{4} - \frac{\gamma_{\prime}^2}{4}\right) \cos(\eta + \eta') \Big\}$$

$$- \frac{3a^2}{8a_{\prime}^3} \left\{ \frac{1}{2} b_{5,0} + b_{5,1} \cos \tau + b_{5,2} \cos 2\tau +, \&c. \right\}$$

$$\left\{ \left(\frac{\gamma^2}{2} + \frac{\gamma_{\prime}^2}{2}\right) \cos \tau - \frac{\gamma^2}{2} \cos(\tau - 2\eta) - \frac{\gamma_{\prime}^2}{2} \cos(\tau + 2\eta_{\prime}) - \gamma\gamma_{\prime} \cos(\eta - \eta_{\prime}) + \gamma\gamma_{\prime} \cos(\eta + \eta_{\prime}) \right\}^2$$

$$\left\{ \left(\frac{\gamma^2}{2} + \frac{\gamma_{\prime}^2}{2}\right) \cos \tau - \frac{\gamma^2}{2} \cos(\tau - 2\eta) - \frac{\gamma_{\prime}^2}{2} \cos(\tau + 2\eta_{\prime}) - \gamma\gamma_{\prime} \cos(\eta - \eta_{\prime}) + \gamma\gamma_{\prime} \cos(\eta + \eta_{\prime}) \right\}^2$$

$$= \left\{ \frac{\gamma^4}{8} + \frac{\gamma^2\gamma_{\prime}^2}{4} + \frac{\gamma_{\prime}^4}{8} \right\} \{1 + \cos 2\tau\} \qquad + \left\{ - \frac{\gamma^4}{4} - \frac{\gamma^2\gamma_{\prime}^2}{4} \right\} \{\cos(2\tau - 2\eta) + \cos 2\eta\}$$

$$+ \left\{ - \frac{\gamma^2\gamma_{\prime}^2}{4} - \frac{\gamma_{\prime}^4}{4} \right\} \{\cos(2\tau + 2\eta_{\prime}) + \cos 2\eta_{\prime}\}$$

$$+ \left\{ - \frac{\gamma^3\gamma_{\prime}}{2} - \frac{\gamma\gamma_{\prime}^3}{2} \right\} \{\cos(\tau + \eta - \eta_{\prime}) + \cos(\tau - \eta + \eta_{\prime})\}$$

$$+ \gamma\gamma_{\prime} \left\{ \frac{\gamma^2}{2} + \frac{\gamma_{\prime}^2}{2} \right\} \{\cos(\tau + \eta + \eta_{\prime}) + \cos(\tau - \eta - \eta_{\prime})\}$$

$$+ \frac{\gamma^4}{8} \{1 + \cos(2\tau - 2\eta)\} + \frac{\gamma^2\gamma_{\prime}^2}{4} \{\cos(2\tau - 2\eta + 2\eta_{\prime}) + \cos(2\eta - 2\eta_{\prime})\}$$

$$+ \frac{\gamma^3\gamma_{\prime}}{2} \{\cos(\tau - \eta - \eta_{\prime}) + \cos(\tau - 3\eta + \eta_{\prime})\}$$

$$+ \frac{\gamma^3\gamma_{\prime}}{2} \{\cos(\tau - \eta + \eta_{\prime}) + \cos(\tau - 3\eta - \eta_{\prime})\} + \frac{\gamma_{\prime}^4}{8} \{1 + \cos(2\tau - 2\eta_{\prime})\}$$

$$+ \frac{\gamma\gamma_{\prime}^3}{2} \{\cos(\tau - \eta + \eta_{\prime}) + \cos(\tau - \eta + 3\eta_{\prime})\} + \frac{\gamma^2\gamma_{\prime}^2}{2} \{1 + \cos(2\eta - 2\eta_{\prime})\}$$

$$- \frac{\gamma^2\gamma_{\prime}^2}{2} \{\cos 2\eta + \cos 2\eta_{\prime}\} + \frac{\gamma^2\gamma_{\prime}^2}{2} \{1 + \cos(2\eta + 2\eta_{\prime})\}.$$

* It is useless to consider these terms, because as R contains no term multiplied by $\frac{a}{a_{\prime}^2}$, if the other part is found to contain any term multiplied by $\frac{a}{a_{\prime}^2}$ it must be neglected, that is to say, got rid of by adding a similar term independent of the quantities b, and with a contrary sign.

In order to obtain the term in R depending upon γ^4, γ^2, $\gamma_{\prime}^2$ and $\gamma_{\prime}^4$, it is sufficient to take

$$R = \frac{m\,a}{a_{\prime}^2}\left\{\frac{1}{2}\,b_{3,0} + b_{3,1}\cos\tau + b_{3,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{-\left(\frac{3}{8}\gamma^2 + \frac{\gamma^2\gamma_{\prime}^2}{8} + \frac{3}{8}\gamma_{\prime}^4\right)\cos\tau + \gamma\,\gamma_{\prime}\,(\gamma^2 + \gamma_{\prime}^2)\cos(\eta - \eta_{\prime}) - \frac{\gamma^2\gamma_{\prime}^2}{8}\cos(\tau - 2\,\eta + 2\,\eta_{\prime}) +, \&c.\right\}$$

$$- \frac{3\,m_{\prime}\,a^2}{8\,a_{\prime}^3}\left\{\frac{1}{2}\,b_{5,0} + b_{5,1}\cos\tau + b_{5,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{\left(\frac{\gamma^2}{2} + \frac{\gamma_{\prime}^2}{2}\right)\cos\tau - \frac{\gamma^2}{2}\cos(\tau - 2\,\eta) - \frac{\gamma_{\prime}^2}{2}\cos(\tau + 2\,\eta_{\prime}) - \gamma\,\gamma_{\prime}\cos(\eta - \eta_{\prime}) + \gamma\,\gamma_{\prime}\cos(\eta + \eta_{\prime})\right\}^2$$

$$= \frac{m_{\prime}\,a}{a_{\prime}^3}\left\{\frac{1}{2}\,b_{3,0} + b_{3,1}\cos\tau + b_{3,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{-\left(\frac{3}{8}\gamma^4 + \frac{\gamma^2\gamma_{\prime}^2}{8} + \frac{3}{8}\gamma_{\prime}^4\right)\cos\tau + \gamma\,\gamma_{\prime}\,(\gamma^2 + \gamma_{\prime}^2)\cos(\eta - \eta_{\prime}) - \frac{\gamma^2\gamma_{\prime}^2}{8}\cos(\tau - 2\,\eta + 2\,\eta_{\prime}) +, \&c.\right\}$$

$$- \frac{3\,m_{\prime}\,a^2}{8\,a_{\prime}^2}\left\{\frac{1}{2}\,b_{5,0} + b_{5,1}\cos\tau + b_{5,2}\cos 2\,\tau +, \&c.\right\}$$

$$\left\{\frac{\gamma^4}{4} + \frac{5\,\gamma^2\gamma_{\prime}^2}{4} + \frac{\gamma_{\prime}^4}{4} - \frac{\gamma\,\gamma_{\prime}}{2}(\gamma^2 + \gamma_{\prime}^2)\cos(\tau + \eta - \eta_{\prime}) - \frac{\gamma\,\gamma_{\prime}}{2}(\gamma^2 + \gamma_{\prime}^2)\cos(\tau - \eta + \eta_{\prime}) + \left(\frac{\gamma^4}{8} + \frac{\gamma^2\gamma_{\prime}^2}{4} + \frac{\gamma_{\prime}^4}{8}\right)\cos\tau + \frac{3}{4}\gamma^2\gamma_{\prime}^2\cos(2\,\eta - 2\,\eta_{\prime}) + \frac{\gamma^2\gamma_{\prime}^2}{4}\cos(2\,\tau - 2\,\eta + 2\,\eta_{\prime})\right\}$$

$$= \left\{-\frac{9\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{9\,a^2}{64\,a_{\prime}^3}\,b_{5,2} - \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2}\right\}\gamma^4$$

$$+ \left\{-\frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2} - \frac{15\,a^2}{64\,a_{\prime}^3}\,b_{5,0} - \frac{15\,a^2}{64\,a_{\prime}^3}\,b_{5,2}\right\}\gamma^2\,\gamma_{\prime}^2$$

$$+ \left\{-\frac{9\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{9\,a^2}{64\,a_{\prime}^3}\,b_{5,2} - \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2}\right\}\gamma_{\prime}^4$$

$$+ \left\{\frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}\right\}\gamma\,\gamma_{\prime}\,(\gamma^2 + \gamma_{\prime}^2)\cos(\tau - \eta + \eta_{\prime})$$

$$+ \left\{-\frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2} - \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{64\,a_{\prime}^3}\,b_{5,2}\right\}\gamma^2\,\gamma_{\prime}^2\cos(2\,\tau - 2\,\eta + 2\,\eta_{\prime})$$

$$= \left\{-\frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}\right\}\gamma^4 + \left\{-\frac{9\,a^2}{32\,a_{\prime}^3}\,b_{5,0} - \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,2}\right\}\gamma^2\,\gamma_{\prime}^2$$

$$+ \left\{-\frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0} + \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,2}\right\}\gamma_{\prime}^4 + \frac{3\,a^2}{16\,a_{\prime}^3}\,b_{5,0}\,\gamma\,\gamma_{\prime}\,(\gamma^2 + \gamma_{\prime}^2)\cos(\tau - \eta + \eta_{\prime})$$

$$- \frac{3\,a^2}{32\,a_{\prime}^3}\,b_{5,0}\,\gamma^2\,\gamma_{\prime}^2\cos(2\,\tau - 2\,\eta + 2\,\eta_{\prime}).$$

In order to give another example of the employment of this method, I propose to calculate the coefficient of

$$e\,\gamma^4 \cos(13\,\tau - \xi - 4\,\eta),$$

the argument of which occurs in Professor AIRY's inequality of Venus, $n\,t$ and $n_{,}t$ being the mean motions of that planet and of the earth.

It is easily seen from the preceding pages that R contains the term

$$-\frac{3}{128}\frac{a^2}{a_{,}^3}\gamma^4\, b_{5,11} \cos(13\,\tau - 4\,\eta).$$

If the coefficient of $\gamma^4 \cos(13\,\tau - 4\,\eta)$ be denoted by R_1

$e\,\gamma^4 \cos(13\,\tau - \xi - 4\,\eta)$ . . . R_3

$$R_3 = -\frac{a\,\mathrm{d}\,R_1}{2\,\mathrm{d}\,a} - 9\,R_1$$

$$= -\frac{3}{128}\left\{-\frac{a^2}{a_{,}^3} b_{5,11} + \frac{5\,a^3}{a_{,}^4}\left(\frac{a}{a_{,}} b_{7,11} - \frac{1}{2} b_{7,10} - \frac{1}{2} b_{7,12}\right) - \frac{9\,a^2}{a_{,}^3} b_{5,11}\right\}$$

$$= \frac{3}{128}\left\{\frac{10\,a^2}{a_{,}^3} b_{5,11} - \frac{5\,a^3}{a_{,}^4}\left(\frac{a}{a_{,}} b_{7,11} - \frac{1}{2} b_{7,10} - \frac{1}{2} b_{7,12}\right)\right\}$$

And R contains the term

$$\frac{3}{128}\left\{\frac{10\,a^2}{a_{,}^3} b_{5,11} - \frac{5\,a^3}{a_{,}^4}\left(\frac{a}{a_{,}} b_{7,11} - \frac{1}{2} b_{7,10} - \frac{1}{2} b_{7,12}\right)\right\} e\,\gamma^4.$$

Professor AIRY has

$$\left\{\frac{27}{8}(0,0) + \frac{3}{16}(0,1)\right\} C_{\frac{5}{2}}^{11}\, e\, f^4. *$$

In Professor AIRY's notation

$$f = \sin\frac{\iota}{2} \qquad f^4 = \frac{\gamma^4}{16} \qquad (0,0)\; C_{\frac{5}{2}}^{11} = \frac{a^2}{a_{,}^3} b_{5,2} \qquad (0,1)\; C_{\frac{5}{2}}^{11} = \frac{2\,a^2}{a_{,}^3} b_{5,11} - \frac{a^2}{a_{,}^3}\frac{\mathrm{d}\,b_{5,11}}{\mathrm{d}\,a}$$

and substituting my notation in Professor AIRY's expression, that which I have found results.

The method I have given of developing the disturbing function in terms of the mean longitudes may also be employed with advantage in procuring the development in terms of the true longitudes. In this problem

$$\frac{\mathrm{d}\,R}{\mathrm{d}\,e} = \frac{r\,\mathrm{d}\,R\,\mathrm{d}\,r}{\mathrm{d}\,r\,r\,\mathrm{d}\,e} = \frac{a\,\mathrm{d}\,R\,\mathrm{d}\,r}{\mathrm{d}\,a\,r\,\mathrm{d}\,e}$$

$$\frac{\mathrm{d}\,r}{r\,\mathrm{d}\,e} = \frac{\mathrm{d}\,.\log.\,r}{\mathrm{d}\,e}$$

$$\log.\,r = \log.\,a + \log.\,(1 - e^2) - \log.\,(1 + e\cos(\lambda - \varpi)$$

$$= \log.\,a - e^2 - \frac{e^4}{2} - e\cos(\lambda - \varpi) + \frac{e^2}{2\,.\,2}\left\{1 + 2\cos(2\,\lambda - 2\,\varpi)\right\}$$

* See p. 89 of Professor AIRY's paper.

$$- \frac{83}{3 \cdot 4} \left\{ 3 \cos (\lambda - \varpi) + \cos (3 \lambda - 3 \varpi) \right\}$$

$$+ \frac{84}{4 \cdot 8} \left\{ 3 + \cos (2 \lambda - 2 \varpi) + \cos (4 \lambda - 4 \varpi) \right\}$$

$$= \log. a - \frac{3 e^2}{4} - \frac{13}{32} e^4 - e \left(1 + \frac{e^2}{4}\right) \cos (\lambda - \varpi) + \frac{e^2}{4} \left(1 + \frac{e^2}{2}\right) \cos (2 \lambda - 2 \varpi)$$

$$- \frac{e^3}{12} \cos (3 \lambda - 3 \varpi) + \frac{e^4}{32} \cos (4 \lambda - 4 \varpi)$$

$$\frac{d r}{r d e} = - \frac{3 e}{2} - \frac{13}{8} e^3 - e \left(1 + \frac{3}{4} e^2\right) \cos (\lambda - \varpi) + \frac{e^2}{2} (1 + e^2) \cos (2 \lambda - 2 \varpi)$$

$$- \frac{e^2}{4} \cos (3 \lambda - 3 \varpi) + \frac{e^3}{8} \cos (4 \lambda - 4 \varpi).$$

It follows from the analysis of M. Poisson, in his Mémoire sur le Mouvement de la Lune autour de la Terre, that the coefficient of $\cos (2 \varpi - 2 \varpi_{\prime})$ in the development of the quantity

$$\frac{r^2 r_{\prime}^2 R}{a^2 a_{\prime}^2 \sqrt{1 - e^2} \sqrt{1 - e_{\prime}^2}}$$

according to the *true longitudes*, is the same as that of $\cos (2 \varpi - 2 \varpi_{\prime})$ in the development of R according to the *mean longitudes*.

If $\frac{r^2 r_{\prime}^2}{a^2 a_{\prime}^2 \sqrt{1-e^2} \sqrt{1-e_{\prime}^2}}$ be called Q,

$$\frac{d Q}{d e} = \frac{a d Q}{d a} \frac{r d r}{d e} \qquad \frac{d Q}{d e_{\prime}} = \frac{a_{\prime} d Q}{d a_{\prime}} \frac{r_{\prime} d r_{\prime}}{d e_{\prime}}.$$

By means of these equations, and after reductions similar to those of which so many examples have been given in the course of this paper, I find the coefficient of

$\cos (2 \lambda - 2 \lambda_{\prime})$ . . . . in Q . . . . $= - a^2 a_{\prime} b_{1,2}$

$e_{\prime} \cos (2 \lambda - 2 \lambda_{\prime} + \lambda_{\prime} - \varpi_{\prime})$ . . . . $= - \frac{a^4}{2 a_{\prime}} b_{3,1} + \frac{a^3}{2} b_{3,2}$

$e e_{\prime} \cos (2 \lambda - 2 \lambda_{\prime} - \lambda + \varpi + \lambda_{\prime} - \varpi_{\prime})$ . . . . $= - \frac{a^3}{4} b_{3,2}$

$e e_{\prime}^2 \cos (2 \lambda - 2 \lambda_{\prime} - \lambda + \varpi + 2 \lambda_{\prime} - 2 \varpi_{\prime})$ . . . $= - \frac{15 a^4}{32 a_{\prime}} b_{5,0} + \frac{3 a^3}{16} b_{5,1} + \frac{9 a^4}{32 a_{\prime}} b_{5,2}$

$e^2 e_{\prime}^2 \cos (2\lambda - 2\lambda_{\prime} - 2\lambda + 2\varpi + 2\lambda_{\prime} - 2\varpi_{\prime})$ or $e^2 e_{\prime}^2 \cos (2\varpi - 2\varpi_{\prime}) = - \frac{9 a^4}{64 a_{\prime}^2} b_{5,2}$.